Peertechz

Annals of Mathematics and Physics

MATHEMATICS AND PHYSICS GROUP

ANNALS OF Mathematics and Physics OPEN ACCESS JOURNAL

ISSN: 2689-7636 DOI: https://dx.doi.org/10.17352/amp

Research Article

Thermodynamic-induced geometry of self-gravitating systems

BI Lev* and AG Zagorodny

Bogolyubov Institute for Theoretical Physics NAS Ukraine, Ukraine

Received: 7 September, 2022
Accepted: 15 September, 2022
Published: 16 September, 2022

***Corresponding author:** BI Lev, Synergetic, Bogolyubov Institute for Theoretical Physics NAS Ukraine, Ukraine, Tel: +038 097 321 87 33; Fax: +038 526 59 98; E-mail: bohdan.lev@gmail.com

ORCID: https://orcid.org/0000-0003-3905-2070

Keywords: Statistical physics; Thermodynamic relation; General relativity, 75.75.Jk, 51.30.+i, 82.50.Lf

Abstract

A new approach based on the nonequilibrium statistical operator is presented that makes it possible to take into account the inhomogeneous particle distribution and provides obtaining all thermodynamic relations of self-gravitating systems. The equations corresponding to the extremum of the partition function completely reproduce the well-known equations of the general theory of relativity. Guided by the principle of Mach's "economing of thinking" quantitatively and qualitatively, is shown that the classical statistical description and the associated thermodynamic relations reproduce Einstein's gravitational equation. The article answers the question of how is it possible to substantiate the general relativistic equations in terms of the statistical methods for the description of the behavior of the system in the classical case.

Introduction

Thermodynamic description of self-gravitational systems based on the quantum theory of gravity at this time does not completely solve the problem. Modern approaches try to offer a quantum geometric model instead of classical differential geometry. The quantum level of description is relevant for extremely small distances, while for large distances it can be defined as ordinary differential geometry and Einstein's gravity [1].

Another point of view is to treat the origin of gravity as a direct manifestation at the macroscopic level of vacuum thermodynamics. This assumption implies that gravity itself is just the low-energy boundary of the known macro physics. In this case, the concepts of density and velocity fields make no sense at the microscopic level and appear only as average values. Similarly to the first approach, this induced gravity picture considers the description of space-time in terms of metric as a phenomenon that is valid for scales greater than some critical length which could be the Planck length. That draws an analogy between the induced gravity and the condensed matter systems [2]. Such approaches give the possibility to determine the gravitational field equation [3-5].

Of particular interest is that the description of the evolution of the horizon can be directly related to the hydrodynamic description of physical fields. Thus, it is possible to establish the correspondence of Rindler's fluid to the derivation of gravitational field equations from local nonequilibrium space-time thermodynamics. From the point of view of statistical thermodynamics, entropy determines the number of independent quantum states compatible with macroscopic parameters. This suggests that Einstein's equation in the presence of the event horizon is analogous to the kinetic equations of transport because they are irreversible and correspond to the description of nonequilibrium thermodynamics, which is analogous to Boltzmann's theorem. The entropy increase theorem can be used for the dynamics of a fluid created by gravity [6-10].

Effective thermodynamics derives from a connection between irreversibility and causality, which is encoded in

Citation: Lev BI, Zagorodny AG (2022) Thermodynamic-induced geometry of self-gravitating systems. Ann Math Phys 5(2): 130-134. DOI: https://dx.doi.org/10.17352/amp.000052

the fundamental concept of entropy, which is appropriately reflected in the description of statistical mechanics. The geometric structure of space-time is introduced into the concept of entropy and the horizon is encoded in the holographic principle [11]. Despite the number of different theoretical approaches, the set of fundamental principles that lead to this assumption is small and very general. It can be supplemented by the concept of space-time construction using quantum entanglement [12].

However, despite the evidence for the validity of such ideas, it is unknown why and how the gravity comes from the degrees of freedom of field theory. Quantitatively and qualitatively, it can be seen that the classical statistical description and the associated thermodynamic relations lead to Einstein's gravitational equation even at the classical level of description. Guided by Mach's principle of "economy of thinking", we will try to obtain the equation of gravity in terms of the statistical approach of classical systems as a result of thermodynamic relations.

This paper proposes a new approach based on the nonequilibrium statistical operator [13], which is more suitable for describing self-gravitational systems with the spatial inhomogeneous distribution. The equations of state and all the necessary thermodynamic characteristics are governed by equations that make the main contribution to the statistical sum. The behavior of the system is regulated by appropriate thermodynamic relations.

The main idea of this work is to provide a detailed description of self-gravitational systems based on the principles of the non-equilibrium statistical operator and to offer a statistical justification of the general theory of relativity. An important result of this approach is the determination of all necessary thermodynamic relations for particle systems with the inhomogeneous spatial distribution. As a result, we can propose, even in the classical approach, to derive the equations of the general theory of relativity from the thermodynamic principle, which is realized by this statistical description. The aim of the work is to find the effective space of thermodynamically stable distribution of self-gravitational systems.

Statistical description of self-gravitating systems

Particle systems interacting over long distances sometimes cannot be described in terms of usual thermodynamic ensembles [14-19] and, therefore, the thermodynamic parameters cannot be completely found by the standard methods of equilibrium statistical mechanics. In particular, if the energy is non-additive, then the canonical ensemble is unsuitable for the study of systems with long-range interactions because the equilibrium states correspond only to local entropy maxima [13,18]. The thermodynamic limit does not exist but the system is stable [20]. But, with many difficulties, the study of particle systems with long-range interaction provides, the development and testing of basic ideas of statistical mechanics and thermodynamics.

Moreover, the methods of the statistical description are insufficiently developed for the case when it is necessary to take into account the spatially inhomogeneous distribution of interacting particles. This concerns primarily the self - gravitational systems. On the other hand, the particle distribution itself determines the geometry of the space where the particles are located [21]. Thus a need arises to determine the geometry of the distribution of matter by the methods of statistical physics.

In our opinion, the most suitable method of describing self-gravitating systems is to employ the nonequilibrium statistical operator [13]. This approach provides a possibility to take into account the spatially inhomogeneous distribution of interacting particles and to follow the evolution of the system. In order to find the thermodynamic functions of the system, we have to use the presentation of the relevant statistical ensembles with allowance for all probable states of this system.

Let us briefly recall the approach [13] that was developed in the papers [22-25] in order to describe the spatially inhomogeneous distribution of interacting particles. Under the assumption that nonequilibrium states of the system can be determined in terms of the energy distribution $H(\mathbf{r})$ and number density of particles $n(\mathbf{r})=\sum_i \delta(\mathbf{r}-\mathbf{r_i})$ the local equilibrium statistical operator distribution may be written in the form [13]:

$$Q_l = \int D\Gamma \exp\left\{-\int\left(\beta(\mathbf{r})H(\mathbf{r})-\eta(\mathbf{r})n(\mathbf{r})\right)d\mathbf{r}\right\} \tag{1}$$

The integration in this formula should be performed over the whole phase space of the system $D\Gamma=\frac{1}{(2\pi\hbar)^3}\prod_i dr_i dp_i$. It should be noted, that in the case of local equilibrium distribution the Lagrange multipliers $\beta(r)$ $\eta(r)$ are functions of spatial points. The phenomenological thermodynamics should take into account the conservation laws for the average values of the physical parameters, i.e., the number of particles $\int n(\mathbf{r})d\mathbf{r}=N$ and energy-momentum $\int H(\mathbf{r})d\mathbf{r}=E$. Hence the definition of a thermodynamic relation for our case of inhomogeneous systems is required. The variation of the statistical operator by the Lagrange multipliers yields the thermodynamic relation given by [13]:

$$-\frac{\delta \ln Q_l}{\delta\beta(\mathbf{r})}=\langle H(\mathbf{r})\rangle_l = E;\frac{\delta \ln Q_l}{\delta\eta(\mathbf{r})}=\langle n(\mathbf{r})\rangle_l = N \tag{2}$$

This relation is the natural extension of the case of equilibrium systems.

The further statistical description requires knowledge of the Hamiltonian of the systems. In the general relativistic case the Hamiltonian of a system of the interacting particle may be written as:

$$H=\sum_i \frac{m_i c^2}{\sqrt{1-\frac{v_i^2}{c^2}}}-\frac{1}{2}\sum_{i,j}W(\mathbf{r_i r_j}) \tag{3}$$

where $W(r_i r_j)$ describes the attractive gravitational interaction

Citation: Lev BI, Zagorodny AG (2022) Thermodynamic-induced geometry of self-gravitating systems. Ann Math Phys 5(2): 130-134.
DOI: https://dx.doi.org/10.17352/amp.000052

and v_i is the particle velocity. For small particle velocities, the energy density of the systems is given by

$$H(\mathbf{r}) = (m(\mathbf{r})c^2 + \frac{p^2(\mathbf{r})}{2m(\mathbf{r})})n(\mathbf{r}) - \frac{1}{2}\int W(\mathbf{r},\mathbf{r}')n(\mathbf{r})n(\mathbf{r}')d\mathbf{r}' \quad (4)$$

This expression may be employed if we divide at the cell level the whole space with different masses and consider the motion in the phase space of an uncompressed liquid, which corresponds to the well-known description of hydrodynamics at the macroscopic level.

In our case of a system with the gravitational character of interaction the nonequilibrium statistical operator may be written in the form

$$Q_l = \int D\Gamma \exp\left\{ -\int\left(\beta(\mathbf{r})\frac{p^2(\mathbf{r})}{2m(\mathbf{r})} + \beta m(\mathbf{r})c^2 - \eta(\mathbf{r})\right)n(\mathbf{r})d\mathbf{r} + \frac{1}{2}\int \beta(\mathbf{r})W(\mathbf{r},\mathbf{r}')n(\mathbf{r})n(\mathbf{r}')d\mathbf{r}d\mathbf{r}' \right\} \quad (5)$$

After mathematical manipulation in terms of the theory of Gauss integrals [26-28] and with the use of an additional field variable φ and the chemical activity $\xi(\mathbf{r}) \equiv \exp\eta(\mathbf{r})$ the statistical operator reduces to the functional integral [25,29]:

$$Q_l = \int D\varphi d\xi \exp - \{f(\varphi(\mathbf{r}), \xi(\mathbf{r}), \beta(\mathbf{r}))\} \quad (6)$$

where the effective "local dimensionless thermodynamic potential" is given by

$$f = \frac{1}{2}\int W^{-1}(\mathbf{r},\mathbf{r}')\varphi(\mathbf{r})\varphi(\mathbf{r}')d\mathbf{r}d\mathbf{r}' - \int \xi(\mathbf{r})\left(\frac{2\pi m(\mathbf{r})}{\hbar^3\beta(\mathbf{r})}\right)^{\frac{3}{2}} \exp\left(\beta(\mathbf{r})m(\mathbf{r})c^2 + \sqrt{\beta(\mathbf{r})}\varphi(\mathbf{r})\right)d\mathbf{r} \quad (7)$$

The inverse operator satisfies the condition $W^{-1}(\mathbf{r},\mathbf{r}'')W(\mathbf{r}'',\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}')$, where the interaction energy is the Green function for this operator. The function $f(\varphi(\mathbf{r}), \xi(\mathbf{r}), \beta(r))$ depends on the distribution of the field variable, the chemical activity and the inverse temperature β(r). The statistical operator makes it possible to obtain the thermodynamic relation that is used in efficient methods developed in the quantum field theory without imposing additional restrictions. Now we apply the saddle point method employed to find the asymptotic value of the statistical operator Q_l for an increasing number of particles N ∞. The dominant contribution is given by the states that satisfy the extreme condition for the functional. It should be noted that the saddle point equation presents the thermodynamic relations and thus we have an equation for the field variables, $\frac{\delta f}{\delta\varphi(\mathbf{r})} = 0$, the normalization condition $\frac{\delta f}{\delta(\eta(\mathbf{r}))} = -\int \frac{\delta f}{\delta\xi(\mathbf{r})}\xi(\mathbf{r})d\mathbf{r} = N$ and also the conservation law for the energy of the system $-\int \frac{\delta f}{\delta\beta(\mathbf{r})}d\mathbf{r} = E$.

The solution of this equation completely determines all macroscopic thermodynamic parameters and describes the general behavior of a system of interacting particles, whether this distribution of particles is spatially inhomogeneous or not. The above set of equations in principle solves the many-particle problem in the thermodynamic limit. The spatially inhomogeneous solution of this equation corresponds to the distribution of interacting particles. Such inhomogeneous behavior is associated with the nature and intensity of the interaction. In other words, the accumulation of particles in a finite spatial region (formation of a cluster) reflects the spatial distribution of the field, chemical activity and temperature. It is a very important notice that only this approach provides a possibility to take into account the inhomogeneous distribution of the temperature and chemical potential that may depend on the spatial distribution of particles in the system.

Thermodynamic relation

For further consideration, we analyze the general presentation of the "local thermodynamic potential" (7). We introduce the new field variables $\sqrt{\beta(\mathbf{r})}\varphi(\mathbf{r}) = \Phi$, then the equation (7) may be written in a simpler form, i.e.,

$$f = \frac{1}{2}\int \beta(\mathbf{r})W(\mathbf{r},\mathbf{r}'))^{-1}\Phi(\mathbf{r})\Phi(\mathbf{r}')d\mathbf{r}d\mathbf{r}' - \int \xi(\mathbf{r})\Lambda^{-3} \exp(\beta(\mathbf{r})m(\mathbf{r})c^2 + \Phi(\mathbf{r}))d\mathbf{r} \quad (8)$$

where the local thermal de-Broglie wavelength is given by $\Lambda(\mathbf{r}) = \left(\frac{\hbar^2\beta(\mathbf{r})}{2m(\mathbf{r})}\right)^{\frac{1}{2}}$. To draw more information about the behavior of the interacting system, we also introduce some new variables. From the normalization condition (9) we obtain

$$\int \xi(\mathbf{r})\Lambda^{-3}\exp(\beta(\mathbf{r})m(\mathbf{r})c^2 + \Phi(\mathbf{r}))d\mathbf{r} = N \quad (9)$$

that yields the macroscopic density whose definition is given by

$$\rho(\mathbf{r}) \equiv \xi(\mathbf{r})\Lambda^{-3}\exp(\beta(\mathbf{r})m(\mathbf{r})c^2 + \Phi(\mathbf{r}))d\mathbf{r} \quad (10)$$

In the case without interaction (for free particles Φ(r)=0), we write the chemical activity in terms of the chemical potential $\xi(\mathbf{r}) = exp(\mu(\mathbf{r})\beta(\mathbf{r}))$ and thus obtain a relation

$$\beta(\mathbf{r})\mu(\mathbf{r}) = \beta(\mathbf{r})m(\mathbf{r})c^2 + \ln(\rho(\mathbf{r})\Lambda^3(\mathbf{r})) \quad (11)$$

that generalizes the relation of the equilibrium statistical mechanics for relativistic systems.

From the minimum "local thermodynamic potential" in terms of new variables, we obtain an equation: given by

$$\int \beta(\mathbf{r})W^{-1}(\mathbf{r},\mathbf{r}')\Phi(\mathbf{r}')d\mathbf{r}' + \rho(\mathbf{r}) = 0 \quad (12)$$

Having multiplied this equation by $\int W(\mathbf{r},\mathbf{r}')d\mathbf{r}'$ we obtain an equation from which the field variables may be found, i.e.,

Citation: Lev BI, Zagorodny AG (2022) Thermodynamic-induced geometry of self-gravitating systems. Ann Math Phys 5(2): 130-134.
DOI: https://dx.doi.org/10.17352/amp.000052

$$\Phi(\mathbf{r})+\int\beta(\mathbf{r})W(\mathbf{r},\mathbf{r}')\rho(\mathbf{r}')d\mathbf{r}'=0 \qquad (13)$$

After that, in the case of relativistic particles with gravitational interaction, we make use of presentation 10 and thus write the general formula for the chemical potential., i.e.,

$$\begin{aligned}\beta(\mathbf{r})\mu(\mathbf{r}) &= \beta(\mathbf{r})m(\mathbf{r})c^2+\Phi+\ln\rho(\mathbf{r})\Lambda^3(\mathbf{r})\\ &= \beta(\mathbf{r})m(\mathbf{r})c^2+\int\beta(\mathbf{r})W(\mathbf{r},\mathbf{r}')\rho(\mathbf{r}')d\mathbf{r}'+\ln(\rho(\mathbf{r})\Lambda^3(\mathbf{r}))\end{aligned} \qquad (14)$$

Now we can determine the average value of the energy of the system using thermodynamic relation

$$\langle H\rangle=-\int\frac{\delta\ln Q_l}{\delta\xi(\mathbf{r})}\frac{\delta\xi}{\delta\beta(\mathbf{r})}\rho(\mathbf{r})d\mathbf{r}=\int\mu(\mathbf{r})\rho(\mathbf{r})d\mathbf{r} \qquad (15)$$

that transforms to the general definition:

$$\begin{aligned}E &= \int\rho(\mathbf{r})\langle H\rangle d\mathbf{r}=\int m(\mathbf{r})c^2\rho(\mathbf{r})d\mathbf{r}\\ &+\int W(\mathbf{r},\mathbf{r}')\rho(\mathbf{r})\rho(\mathbf{r}')d\mathbf{r}'+\int kT\rho(\mathbf{r})\ln(\rho(\mathbf{r})\Lambda^3(\mathbf{r}))d\mathbf{r}\end{aligned} \qquad (16)$$

The last part of this relation is exactly equal to the entropy of the systems and thus the free energy of the system is given by [30],

$$F=\int m(\mathbf{r})c^2\rho(\mathbf{r})d\mathbf{r}+\int W(\mathbf{r},\mathbf{r}')\rho(\mathbf{r})\rho(\mathbf{r}')d\mathbf{r}' \qquad (17)$$

Thus we have all the thermodynamic relations needed for the description of the dynamical behavior of a non-equilibrium system [31]. Such presentation determines the evolution of a non-equilibrium system in the general case, however, the description is possible to describe only in the special case [22,24,25,29].

Thermodynamically induced geometry of self-gravitating systems

The physical theory is based on the postulated geometric properties of the space where the interacting particle are located. The problem of geometry as a whole is equivalent to the problem of the behavior of the fields that form this space [32-35]. In what follows we propose a geometric description of a thermodynamically stable distribution of a self-gravitating system. The character and intensity of the interaction in the system determine the effective geometry of the medium that is provided by the minimum of the total free energy [21]. This article proposed a geometric description of a thermodynamically stable distribution of different interacting particles. The character and intensity of interaction between particles determine the effective geometry of the medium which is provided by the minimum of the total free energy. In the given article is proposed another way to determine the geometry of the self- gravitating system.

For zero temperature free energy is equal to action in the term of density [36]. We can remind that after the Wick transformation the integral of the free energy over time is reduced to the action in the Minkowski space, i.e.,

$$S=\frac{1}{c}\int m(\mathbf{r})c^2\rho(\mathbf{r})d\mathbf{r}dct+\frac{1}{c}\int W(\mathbf{r},\mathbf{r}')\rho(\mathbf{r})\rho(\mathbf{r}')d\mathbf{r}'dct \qquad (18)$$

From the virial theorem [31] we may conclude that the first part of the free energy 17 is the average value of the energy-momentum tensor of free particles $\int\langle T_{ii}\rangle d(\mathbf{r})$. In the relativistic theory [31,37] the energy-momentum tensor for a macroscopic system is given by

$$T_{i,j}=(\epsilon+P)u_iu_j-P\delta_{ij} \qquad (19)$$

where u_i is the four-velocity with the condition $u_iu^i=1$, ϵ is the energy density, and P is the pressure in the system. As is shown in [31,37], the first part of the action determines the energy-momentum tensor $T_{ii}=SpT_{ij}=\epsilon-3P$ in the four-dimensional Euclidean space that may be rewritten in a different four-dimensional curved space provided we take into account the volume element $d\Omega=\sqrt{-g}d^4x$ with g=det g_{ij} where g_{ij} is the metric tensor. After that, we should take into account the part of the energy that is spent on the distribution of matter that forms the geometry [21], i.e.,

$$S_g=\frac{1}{c}\int W(\mathbf{r},\mathbf{r}')\rho(\mathbf{r})\rho(\mathbf{r}')d\mathbf{r}'dct\simeq-\frac{c^2}{32\pi G}\int Rd\Omega \qquad (20)$$

where R is the curvature of the space and $d\Omega=\sqrt{-g}d^4x$ - is the standard form of an element of the four-dimensional volume g=det g_{ij} with the metric tensor g_{ij}. Variation action by matric tensor yields an equation for the gravitational field under the assumption that the curvature is induced by the distribution of matter with the energy-momentums tensor T_{ij}. In the correspondent general theory of relativity, the Einstein equation for the curvature may be obtained in the well-known form given by

$$R_{ij}-\frac{1}{2}Rg_{ij}=\frac{8\pi G}{c^4}T_{ij} \qquad (21)$$

This equation for the gravitational field is incomplete because along with the second-rank tensor R_{ij} another second-rank tensor g_{ij} can be present. A simple linear theory may contain a simplistic linear combination in the form $R_{ij}+\alpha g_{ij}=\beta T_{ij}$. From such a simple reason we can take into account the probable part associated with the symmetry conservation of the tensor. Such native tensor in this case is the metric tensor. This possibility is realized in the Einstein equation with the cosmological constant $\Lambda=V(\phi)$ where the potential depends on the fundamental scalar field ϕ [36]. From this equation, we may conclude that in the dynamics of the Universe the crucial contribution is associated only with the gravitational force while any other forces do not take part in such dynamics. We note again that the distribution of matter determines the geometry. Without this distribution, we cannot speak about any geometry.

Citation: Lev BI, Zagorodny AG (2022) Thermodynamic-induced geometry of self-gravitating systems. Ann Math Phys 5(2): 130-134.
DOI: https://dx.doi.org/10.17352/amp.000052

Conclusion

A new approach that employs a nonequilibrium statistical operator with allowance for inhomogeneous distributions of particles is proposed. This method applies the saddle-point procedure in order to find the dominant contributions to the partition function and provides a possibility to obtain all the thermodynamic parameters of the system. This approach makes it possible to solve the problem with inhomogeneous distributions of particles. On much longer timescales, the evolution towards the true thermal equilibrium is postulated. In this way, we can solve the complicated problem of the statistical description of systems with gravitational interaction.

An attempt is made to find motivation for the evolution of self-gravitating systems. The change of the local thermodynamic potential induces the inhomogeneous distribution of matter and determines the geometry of this distribution. The thermodynamically induced distribution of matter completely determines the geometry of the latter. This interpretation of the geometry makes it possible to follow the evolution of the system. The paper answers the question of how is it possible to substantiate the general relativistic equations in terms of the statistical methods for the description of the behavior of the system in the classical case.

Acknowledgment

This work was partially supported by the Target Program of Fundamental Research of the Department of Physics and Astronomy of the National Academy of Sciences of Ukraine “Noise-induced dynamic and correlation in non-equilibrium systems” (No 0120U101347).

References

1. Chirco G. Thesis, International School for Advanced Studies. 2011.
2. Barcel C, Liberat S, Visser M. Analogue Gravity. Living Rev Relativity. 2011; 14: 3. http://www.livingreviews.org/lrr-2011-3
3. Jacobson T. Thermodynamics of spacetime: The Einstein equation of state. Phys Rev Lett. 1995 Aug 14;75(7):1260-1263. doi: 10.1103/PhysRevLett.75.1260. PMID: 10060248.
4. Jacobson T, Parentani R. Horizon Entropy. Found Phys. 2003; 235: 323.
5. Jacobson T. Introductory Lectures on Black Hole Thermodynamics. 2005. www.physics.umd.edu/grt/taj/776b/lectures.pdf.
6. Padmanabhan T. Classical and quantum thermodynamics of horizons in spherically symmetric spacetimes. Class Quan Grav. 2002; 19: 5387.
7. T. Padmanabhan. Thermodynamical aspects of gravity: new insights. Rept Prog Phys. 2010; 73: 046901.
8. Susskind L, Uglum J. Black hole entropy in canonical quantum gravity and superstring theory. Phys Rev D. 1994; 50: 2700.
9. Susskind L. The world as a hologram. J Math Phys. 1995; 36: 6377.
10. Susskind L, Witten E. The Holographic bound in anti-de Sitter space. 1998. arXiv/hep-th/9805114.
11. Verlinde E. On the Origin of Gravity and the Laws of Newton. Journal of High Energy Physics. 2011; 8: 137. 10.1007/JHEP04(2011)029.
12. Raamsdonk MV. Building up spacetime with quantum entanglement. Gen Rel Grav. 2010; 42: 2323-2329.
13. Zubarev DN. Non-equilibrium statistical thermodynamics (Consultans Bareu, New York), 1974.
14. Thirring W. Systems with negative specific heat. Z Phys. 1970; 235: 339.
15. Chandrasekhar S. An introduction to the study of stellar structure. New York: Dover Publications. 1942.
16. Chavanis PH, Rosier C, Sire C. Thermodynamics of self-gravitating systems. Phys Rev E Stat Nonlin Soft Matter Phys. 2002 Sep;66(3 Pt 2A):036105. doi: 10.1103/PhysRevE.66.036105. Epub 2002 Sep 10. PMID: 12366182.
17. Beheshti S, Normann M, Valiente Kroon JA. Future stability of self-gravitating dust balls in an expanding universe. Phys Rev D. 2022; 105: 124027. https://doi.org/10.1103/PhysRevD.105.124027
18. Ourabah K. Superstatistics: Consequences on gravitation and cosmology. Phys Rev D. 2020; 102: 043017. https://doi.org/10.1103/PhysRevD.102.043017
19. Wren AJ. The Fifteenth Marcel Grossmann Meeting. 1316-1319. https://doi.org/10.1142/9789811258251
20. Laliena V. Nuclear Physics B. 2003; 668: 403.
21. Rebesh AP, Lev BI. Phys Lett A. 2017; 381: 2538. DOI:o5.0510375-9601
22. Lev BI. Nonequilibrium self-gravitating system. International Journal of Modern Physics B. 2011; 25: 2237.
23. Lev BI. Statistical Derivation of the Fundamental Scalar Field. Journal of Modern Physics. 2018; 9: 2223.
24. Lev BI. Zagorodny AG. Statistical description of Coulomb-like systems. Phys Rev E. 2011; 84: 061115.
25. Lev BI. Statistical Induced Dynamic of Self-Gravitating System. Journal of Modern Physics. 2019; 10: 687.
26. Kleinert H. Gauge Field in Condensed Matter, Word Scientific, Singapure, 1989.
27. Hubbard J. Calculation of Partition Functions. Phys Rev Lett. 1959; 3: 77.
28. Stratonovich RL, Sov. Phys. Dokl. 1958; 2: 416
29. B. I. Lev and A. Ya. Zhugaevych. Statistical description of nonequilibrium self-gravitating systems. 1998; 57: 6460
30. Feynman RP. Statistical Mechanics: A Set Of Lectures (Advanced Books Classics) Paperback - 26 March 1998.Statistical Mechanics, California Institute of Technology.1972.
31. Landau LD, Lifshiz EM. Classical and quantum mechanics of the damped harmonic oscillator. Field theory (Pergamon, London). 1981.
32. Hilbert D. The Basics of Physics. Die Grundlagen der Physik, Koeniglichen Ges. Wiss. Goett., Math.-Phys. Kl. Nachr. 1915; 395.
33. Einstein A. An Extended Newtonian Theory for Gravitational Bound Systems. Ann. Phys. 1916; 354: 769.
34. Weinberg S. Recent progress in gauge theories of the weak, electromagnetic and strong interactions. Rev. Mod. Phys. 1974; 46: 255.
35. Wu TT, Yang CN, in: Mark H, Fernbach S(Eds.), Properties of Matter under Unusual Conditions, Wiley-Interscience. New York. 1969; 349.
36. Linde AD. Elementary particle physics and inflationary cosmology. Horwood Academic. Switzerland, 1990.
37. Zeldovich, Ya B, Novikov ID. Theory of gravitation and the evolution of stars. Nauka, Moskva. 1971. nces

Citation: Lev BI, Zagorodny AG (2022) Thermodynamic-induced geometry of self-gravitating systems. Ann Math Phys 5(2): 130-134.
DOI: https://dx.doi.org/10.17352/amp.000052