

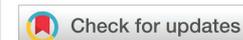
**Received:** 29 May, 2023  
**Accepted:** 16 June, 2023  
**Published:** 19 June, 2023

\*Corresponding author: Ling Xie, Dongkou Bamboo City Center Hospital in Hunan Province, China, E-mail: 29997609@qq.com, xieling1968@hotmail.com

**Keywords:** Limitless hotel; Cantor; Limited; Limitless

**Copyright License:** © 2023 Xie L. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

<https://www.peertechzpublications.com>



Letter to Editor

# The conclusion of the limitless hotel problem

Ling Xie\*

Dongkou Bamboo City Center Hospital in Hunan Province, China

## Abstract

Mathematician Cantor's Set theory appeared paradox and mathematical theory crisis. The famous German mathematician Hilbert used "Hilbert Hotel" to describe Cantor's Set theory paradox. At that time, people could not find a strict mathematical theory to refute Cantor's Set theory, but let everyone get used to and accept Cantor's Set theory, and thought that it was not a paradox. After the proposal of the limitless Hotel question, it caused controversy between the two parties.

I quoted the definition of mathematical logic and got the correct answer.

Proved that there is no paradox in limitless hotels (Reason: limitless hotels cannot increase the number of new guests staying.).

A deep analysis of Cantor's limitless elements and the infeasibility of one-to-one correspondence was conducted.

**2020 Mathematics Subject Classification:** 03G27, 03F07, 03D45, 03F55

## The limitless hotel problem can only have one conclusion

In 1924, the famous German mathematician Hilbert (1862-1943) [1] proposed the famous "Hilbert Hotel" problem in a speech.

**Infinite hotel [2]:** Infinite Hotel has limitless rooms. It is stipulated that each room can accommodate only one person. Cantor [1] said: new guests can also be accommodated.

In order to prevent the two factions from arguing {finite and infinite}, I simplified limitless Hotel as any one room in the hotel is occupied, and only one person is allowed. Question: Can we increase the number of new guests?

**Prove:**

Still the old method, first make a logical definition [3] and define it as an exclusive symbol.

Proof can only be based on defined exclusivity symbols.

Only in this way can disputes and ambiguity be prevented.

{empty room} definition:  $\Sigma (= 0)$

{Any one room has people, and only one person is allowed to live} Definition:  $\forall \Sigma (= 1)$

{New guests who can stay} Definition:  $\{\exists \Sigma (= 0)\}$

$\therefore \forall \Sigma (= 1)$

$\therefore \{\forall \Sigma (= 1)\} \not\equiv \{\Sigma (= 0)\}$

$\therefore \{\forall \Sigma (= 1)\} \not\equiv \{\exists \Sigma (= 0)\}$

(QED).

Comment:



Note: limitless hotels have set up the premise that there are no vacant rooms, so there is no need to assume the possibility of empty rooms.

Because being able to accommodate new guests depends on having a "vacant room".

It is a wrong viewpoint to use the concept of infinity to accommodate new guests.

It proves that there is a contradiction in limitless hotels that can increase new customers.

In fact, the limitless Hotel also violates the pigeonhole principle, but Cantor countered that the pigeonhole principle is aimed at the concept of finite, and the limitless Hotel belongs to the concept of infinite. So the second section is to prove that the one-to-one correspondence of limitless concept applications is incomplete.

### The one-to-one correspondence between limitless concepts is incomplete

The limitless hotel problem involves the mathematical concept of one-to-one correspondence.

Firstly, define the concepts applied.

Arguments without defined concepts can only lead to arguments between both parties.

Define one-to-one correspondence, but it must also be defined that it cannot be a one-to-one correspondence.

**Prove:**

First, make a logical definition and define it as an exclusive symbol.

Definition of one-to-one correspondence between A and B:  $\frac{A}{B}$ .

A cannot achieve one-to-one correspondence definition:

$$\frac{\square}{A}, \text{ or } \frac{A}{\square}$$

Limitless arrangement of elements ( $\alpha$ ): {1,2,3,4,5,6....}

Limitless positive even element arrangement ( $\beta$ ): {2,4,6,8,10,12....}

{ $\alpha, \beta$ }  $\Rightarrow$  The first method can achieve one-to-one correspondence: (I): { $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \dots$ }

$\therefore$  (I) Realized one-to-one correspondence:

$$\{ \frac{\square}{A} \in (I) \in \frac{A}{B}, \frac{A}{\square} \notin (I) \}$$

{ $\alpha, \beta$ }  $\Rightarrow$  It can be proven that the second method cannot achieve one-to-one correspondence:

$$(II): \{ \frac{\square}{2}, \frac{1}{4}, \frac{2}{6}, \frac{3}{8}, \frac{4}{10}, \dots \}$$

Prove:

$$\therefore (III): \{ \frac{1}{4}, \frac{2}{6}, \frac{3}{8}, \frac{4}{10}, \frac{5}{12}, \dots \}$$

$\therefore$  (III) Realized one-to-one correspondence:

$$\frac{A}{\square} \notin (III) \in \frac{A}{B}, \frac{\square}{A} \notin (III)$$

$$\therefore (II) \ni \{ \frac{\square}{2} + (III) \}$$

$$\therefore \frac{A}{\square} \notin (III) \in \frac{A}{B},$$

{Any number in (III) has completed a one-to-one correspondence, and no more numbers ( $\frac{A}{\square}$ ) corresponding to ( $\frac{\square}{2}$ ) can be found to complete: one-to-one correspondence

$$\} \nRightarrow \{ \frac{\square}{2} + \frac{A}{\square} = \frac{A}{B} \}$$

$$\Rightarrow \{ \frac{\square}{2} \neq 0 \}: \text{Yes } (\frac{\square}{2}) \text{ Eternal}$$

$$\therefore (II) \ni \frac{\square}{2} \in \frac{\square}{A}$$

$\therefore$  (II) cannot achieve one-to-one correspondence.

$$\Rightarrow \{ (\alpha, \beta) \Rightarrow \frac{A}{B}, \frac{\square}{A} \leftarrow (\alpha, \beta) \nRightarrow \frac{A}{B} \},$$

The concept of 'infinity'. There will be two opposite conclusions:

One-to-one correspondence, but not one-to-one correspondence.

(QED).

Ludwig Wittgenstein had particular doubts about limitless operations [4].

I have proven through the mathematical theory that Ludwig Wittgenstein was correct in his special suspicion of limitless operations.

Why is the one-to-one correspondence between limitless concepts incomplete?

This is a deep problem: nonnumbers, Numbers, Limited, limitless.

After defining the concept of "nonnumbers, Number, Limited, limitless "in my other manuscript, I will get a complete answer.

### Logic will not contradict itself

Russell [5] pursued logical consistency throughout his life, and I used mathematical language to prove that this logic would not contradict itself, turning a philosophical problem into a mathematical form.

Prove:

Definition of logic: { $\mathcal{A} \not> \mathcal{A}$ }.

Paradox definition: { $\mathcal{A} > \mathcal{B}, \mathcal{B} > \mathcal{A}$ }



Theorem: Logic will not contradict itself.

$$\{A > B, B > A\} \notin \{A \not> A\}$$

$$\therefore \{A \not> A\}$$

Assumptions:  $\{A > B, B > A\}$

$$\therefore A > B$$

$$\therefore B > A$$

$$\therefore A + B > A + B$$

$$\rightarrow \{A > A\}$$

$\{A > A\}$  conflicts with  $\{A \not> A\}$

$$\therefore \{A > A\} \in \{A > B, B > A\} \notin \{A \not> A\}$$

(QED).

Mathematical and Physical Meaning: Correct theories do not conflict, and two conflicting theories cannot all be correct.

The definition of truth: a theory that conforms to logic.

$\therefore$  A theory that conforms to logic is truth.

Mathematical significance: There is no contradiction and truth that coexist in the mathematical system, So Cantor's infinite theory introducing one-to-one correspondence is incorrect.

### Case: $(A \not> A)$ Proves that perpetual motion machines do not exist

It seems like a purely physical theory. This is also a mathematical logic problem.

The concept of perpetual motion machine [6]: Without external force,  $A$  outputs  $k(k > 0)$  and maintains at least its original  $A$  state.

Definition of perpetual motion machine:

$$A - k(k > 0) \geq A$$

Proof:

$$\therefore A - k(k > 0) \geq A$$

$$\therefore A \geq A + k(k > 0)$$

$$\Rightarrow A > A \text{ (This is the definition of contradiction)}$$

(QED).

This proof is the charm of logical definition.

Don't use the laws of physics to repeatedly argue, and directly use mathematical methods to negate perpetual motion machines.

## Conclusion

Purpose of mathematics and science: It is necessary to give a definite conclusion.

I quoted the definition of Mathematical logic as the basis to prove the following conclusions:

- (1). No matter it is a limitless room or an unlimited room, as long as any room is occupied by one person (and only one person), new guests cannot be accommodated.
- (2). This proves that Cantor's concept of limitless access to mathematics is incomplete. It is not related to whether the room is finite or limitless.
- (3). Without vacant rooms, new guests cannot be accommodated.

## Thinking

What causes the one-to-one correspondence between limitless concepts to be incomplete?

Usually, when people see the application of one-to-one correspondence in the concept of limitless (see 1 method arrangement), they believe that it is correct, Therefore, they will assume that there will be no conflict.  $\rightarrow$  Here, we have overlooked a premise: the concept of limitless must be a number in order to apply the concept of one-to-one correspondence. If the concept of limitless is not a number, applying the concept of one-to-one correspondence will inevitably lead to contradictions.

Because the concept of one-on-one correspondence hides the concept of two numbers being equal and the definition of nonnumbers cannot be Mathematical analysis.

After defining the concepts of "nonnumber, number, finite, limitless" in my other manuscript, I used the definition as a condition to prove that the concept of limitless belongs to nonnumber.

## Statement

- The author has no relevant financial or non-financial interests to disclose.
- The author has no conflict of interest related to the content of this article.
- All authors warrant that they have no relationship with any organization or entity or are involved in any subject or material of financial or non-financial interest discussed herein.
- The author has no economic or proprietary interest in any of the materials discussed herein.
- Funding (supported by the author).
- Publication consent (consent)



The author (Xie Ling) submits a manuscript to the Yearbook of Mathematics and Physics.

### Thank

Thank you to nature for allowing me to discover logical definition methods and avoid circular definitions.

Thank you to nature for giving me the perseverance to pursue truth {Definition of truth:  $(A \not\sim A)$ ; Definition of contradiction:  $(A > A)$ }.

Thank you to the Mathematics and Physics Yearbook for providing me with a presentation platform.

Thank you to the peer reviewer of the Mathematics and Physics Yearbook for your hard work.

**Regarding financial information:** There is no sponsoring unit, self-funded publication.

### References

1. Singh S. The never-ending story. Nature. 2005; 434:437–438.
2. Ellis GFR, Meissner KA, Nicolai H. The physics of infinity. Nature Physics. 2018; 14: 770-772.
3. Ling X. How to define the six standards. Annals of Mathematics and Physics. 2022; 5: 29. <https://www.peertechzpublications.com/articles/AMP-5-137.pdf>
4. Carr HW. Tractatus Logico-Philosophicus. Nature. Books & Arts. 24 Feb 1923; 111: 246-247.
5. Whittaker ET. The Philosophy of Bertrand Russell. Nature. Books & Arts 03 Feb 1945; 155: 128-131.
6. Highton H. Perpetual motion machine. Research 09 Mar 1871. Nature. 3: 368-369.

Discover a bigger Impact and Visibility of your article publication with Peertechz Publications

#### Highlights

- ❖ Signatory publisher of ORCID
- ❖ Signatory Publisher of DORA (San Francisco Declaration on Research Assessment)
- ❖ Articles archived in worlds' renowned service providers such as Portico, CNKI, AGRIS, TDNet, Base (Bielefeld University Library), CrossRef, Scilit, J-Gate etc.
- ❖ Journals indexed in ICMJE, SHERPA/ROMEO, Google Scholar etc.
- ❖ OAI-PMH (Open Archives Initiative Protocol for Metadata Harvesting)
- ❖ Dedicated Editorial Board for every journal
- ❖ Accurate and rapid peer-review process
- ❖ Increased citations of published articles through promotions
- ❖ Reduced timeline for article publication

Submit your articles and experience a new surge in publication services (<https://www.peertechz.com/submission>).

Peertechz journals wishes everlasting success in your every endeavours.