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Research Article

# A new reduced quantile function for generating families of distributions

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## Abstract

In this paper, a variant of the *T-X(Y)* generator was developed by suppressing the scale parameter of the classical Lomax distribution in the quantile function. Uniquely, the reduction of the number of parameters essentially accounts for the parsimony of the attendant model. The study considered the Exponential distribution as the transformer and consequently obtained the New Reduced Quantile Exponential-G (NRQE-G) family. The Type-II Gumbel distribution was deployed as the baseline to obtain a special sub-model known as the New Reduced Quantile Exponential Type-II Gumbel (NRQE-T2G) model. Some functional properties of the distribution namely, moment and its related measures such as the mean, variance, second, third, and fourth moments were obtained. The Mode, skewness, Kurtosis, index of dispersion, coefficient of variation, order statistics, survival, hazard, and quantile function were also derived. The maximum likelihood estimation method was used to estimate its parameters. The model's credibility, applicability, and flexibility were proven using two real-life datasets.

## The genesis

In modeling real-world phenomena using probability distributions, researchers have developed several models that fit various scenarios. Motivated by robust statistical inferences, methods for developing univariate continuous probability distributions and families of such continuous distributions have been suggested by many authors. While some of the methods require mathematical rigor others such as the mixing of two-component distributions first deployed by Lindley [1] and its classes Onyekwere and Obulezi [2], Onyekwere, et al. [3], Tolba, et al. [4], Etaga, et al. [5] and Obulezi, et al. [6] are trivial. Pearson [7] early work on Leibnitz's theorem for generating probability values is remarkable at the time. This method is popularly known as the method of differential equations. McDonald [8] introduced the generalized beta distribution of the first and second (GB1 and GB2 respectively) and its variant by McDonald and Xu [9] with special cases such as GB1 and GB2. Azzalini [10] developed the skew-normal distribution with a skew parameter $\lambda$. Skew-symmetric distributions were investigated by Kotz and Vicari [11] with a detailed framework for generating families of skewed distributions proposed by Ferreira and Steel [12]. Marshall and Olkin [13] proposed a method of introducing an additional parameter to a baseline distribution using its survival function.

The Beta generated family of distributions introduced by Eugene, Lee, and Famoye [14] forms a basis for many generators of families of distributions in the statistical literature today. The Cumulative Distribution Function (CDF) of the Beta-G family is

$$G(x) = \int_0^{F(x)} b(t)\,dt, \quad (1)$$



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Where $F$ is the CDF of any random variable and $b$ is the generator. For continuous $X$, the associated Probability Density Function (PDF) to Eq. 1 is

$$g(x)=\frac{f(x)}{B(\alpha,\beta)}F^{\alpha-1}(x)(1-F(x))^{\beta-1};\quad \alpha>0,\beta>0. \tag{2}$$

Alzaatreh, Lee, and Famoye [15] introduced a method of generating continuous univariate distributions and their families by replacing the beta pdf in eq 1 with PDF $m(t)$ of any continuous random variable $T$ and deploying the function $Z\{F(x)\}$ that satisfies the following conditions;

- $Z\{F(x)\}\in[a,b]$.
- $Z$ is differentiable and monotonically non-decreasing.
- $Z\{F(x)\}\to a$ as $x\to-\infty$ and $Z\{F(x)\}\to b$ as $x\to\infty$,

Where $[a,b]$ is the domain of the random variable $T$ such that $-\infty\le a<b\le\infty$. The above argument led to the interesting generator known as the $T$-$X\{Z\}$ generator of families of distributions with CDF given as follows;

$$G(x)=\int_a^{Z\{F(x)\}}m(t)dt=M\{Z(F(x))\}, \tag{3}$$

Where $R$ is the CDF of the random variable $T$, which is the transformer. The PDF associated with eq 3 is

$$g(x)=\left\{\frac{d}{dx}Z(F(x))\right\}m\{Z(F(x))\}. \tag{4}$$

Ideally, for discrete random variables $X$, the resulting distribution is also discrete.

Aljarrah, Lee, and Famoye [16] developed a wider class of $Z$ functions with the following definition. Let $Z:(0,1)\to(a,b)$, for $-\infty\le a<b\le\infty$, be a right-continuous and non-decreasing function such that $\lim_{u\to0^+}Z(u)=a$ and $\lim_{u\to1^-}Z(u)=b$, then $G(x)=M\{Z(F(x))\};x\in(-\infty,\infty)$ is a distribution function provided the following conditions are satisfied;

- $G(x)$ is non-decreasing.
- $G(x)$ is right-continuous.
- $G(x)\to0$ as $x\to-\infty$ and $G(x)\to1$ as $x\to\infty$.

Succinctly, let the domain of $T$ be $(a,b)$ and let $\Delta$ be the CDF of a random variable say $Y$ assuming values on $(a,b)$ and define the quantile function of $\Delta$ by

$$G(x)=M\{Z(F(x))\};x\in(-\infty,\infty) \tag{5}$$

If $\Delta$ is continuous and strictly increasing, then $Q_Y=\Delta^{-1}$ is continuous and strictly increasing (Shorack and Wellner [17]). Taking $Z$ to be the quantile function of a strictly non-decreasing distribution function $\Delta$ for the random variable $Y$ written as $Z(u)=Q_Y(u)$, $u\in(0,1)$, the $Q_Y$ is continuous and non-decreasing with cdf of a $T$-$X\{Y\}$ family using the quantile function $Q_Y$ can be expressed as

$$G(x)=\int_a^{Q_Y(F(x))}m(t)dt=M\{Q_Y(F(x))\};\quad x\in(-\infty,\infty), \tag{6}$$

Where $Y$ has a density function $\delta(y)>0,\forall y$ in a neighborhood of $Q_Y(u)$, for $u\in(0,1)$, then $\frac{d}{dx}Q_Y(u)$ exists and equals $\left[\delta(Q_Y(u))\right]^{-1}$ Therefore, the corresponding pdf is given as

$$g(x)=\frac{f(x)}{\delta\{Q_Y(F(x))\}}m\{Q_Y(F(x))\}. \tag{7}$$

**Definition 1.1** (Parsimony) The class $Z$ defined by Aljarrah, Lee, and Famoye [16] produces families of distributions and consequently, distributions with too many parameters because the quantile function of a distribution is a function of the parameters of that distribution. The principle of parsimony therefore entails that a model with fewer parameters is preferred over models with more parameters, provided the models fit the data similarly well.



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With the understanding of the principle of parsimony, this study is therefore motivated by the need to;

1. proposed reduced quantile functions better than those introduced by Aljarrah, Lee, and Famoye [16].

2. based on the reduced quantile function above, parsimonious $T$-$X\{Y\}$ generators of families of continuous univariate distributions are developed.

3. Consequently, guarantee the mathematical tractability of the distributions developed,

4. Maintain the goodness of fit of the distributions and

5. Preserve the flexibility in the application of the distributions.

The scale parameter is redefined as λ = 1 in the quantile function of the Lomax distribution credit to [18]. The resulting function is known as a reduced quantile function given as $Q_Y^r(F(x)) = [1-F(x)]^{-\frac{1}{\alpha}} - 1$ , in this sense, a function of the cdf of any baseline distribution $F(x)$. Suppose the distribution Exponential Distribution is deployed as the transformer with CDF and PDF given as $R(t) = 1 - e^{-t\lambda}$ and $r(t) = e^{-t\lambda}\lambda$ respectively. The CDF and PDF of the new family of distributions named the New Reduced Quantile Exponential-G family (NRQE-G) are given by

$$G(x) = 1 - e^{-\lambda\{[1-F(x)]^{-\frac{1}{\alpha}}-1\}} \tag{8}$$

$$g(x) = \frac{\lambda f(x) e^{-\lambda\{[1-F(x)]^{-\frac{1}{\alpha}}-1\}}}{\alpha[1-F(x)]^{\frac{\alpha-1}{\alpha}}} \tag{9}$$

## Special submodel

A New Reduced Quantile Exponential - Type II Gumbel (NRQE-T2G) Distribution was formed and has pdf and cdf given as:

$$g(x) = \frac{\lambda\theta\beta x^{-\theta-1} e^{-\{\beta x^{-\theta} + \lambda[\{1-e^{-\beta x^{-\theta}}\}^{-\frac{1}{\alpha}}-1]\}}}{\alpha[\{1-e^{-\beta x^{-\theta}}\}^{\frac{1+\alpha}{\alpha}}]}; \quad x \geq 0, \quad \alpha,\beta,\theta,\lambda > 0 \tag{10}$$

and

$$G(x) = 1 - e^{-\lambda[\{1-e^{-\beta x^{-\theta}}\}^{-\frac{1}{\alpha}}-1]}; \quad x \geq 0, \quad \alpha,\beta,\theta,\lambda > 0 \tag{11}$$

The Survival Function of (NRQE-T2G) distribution is given as:

$$S(x,\alpha,\beta,\theta,\lambda) = e^{-\lambda[\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}}-1]}; \quad x \geq 0, \quad \alpha,\beta,\theta,\lambda > 0 \tag{12}$$

The Hazard Rate of The New Reduced Quantile Exponential - Type II Gumbel Distribution $\mathrm{H}(x) = \frac{F(X)}{S(x)}$ , (The CDF of New Reduced Quantile Exponential - Type II Gumbel Distribution), which is given as:

$$H(x) = \frac{\lambda\theta\beta x^{-\theta-1} e^{-\beta x^{-\theta}}}{\alpha[\{1-e^{-\beta x^{-\theta}}\}^{\frac{1+\alpha}{\alpha}}]} \tag{13}$$

The plots of the PDF, CDF, survival function and hazard rate function for hypothetical values are displayed in Figures 1-4. There is an obvious shift from normality as can be observed from the PDF in Figure 1. The CDF survival functions in Figures 2,3 are opposite in shape. While the CDF is increasing, the survival function decreasing. This situation is common in reality depicting the possible extinction of life. The hazard function is popularly used to describe lifetime events. In Figure 4, it is clear that the proposed submodel NRQE-T2G is both increasing, decreasing, and bath-tube-shaped, hence lending the model to a wide spectrum of applications.

3D plots provide better visualization and aid the interpretation of graphs. The 3D plot in Figure 5 represents the PDF of the proposed NRQE-T2G. With $45^{o}$ angular rotation of the plot, the curve does not represent normality hence the model better describes skewed data. Similarly, the 3D plots of the hazard function in Figure 6 depict both increasing and decreasing views with regard to the top and bottom vertexes.



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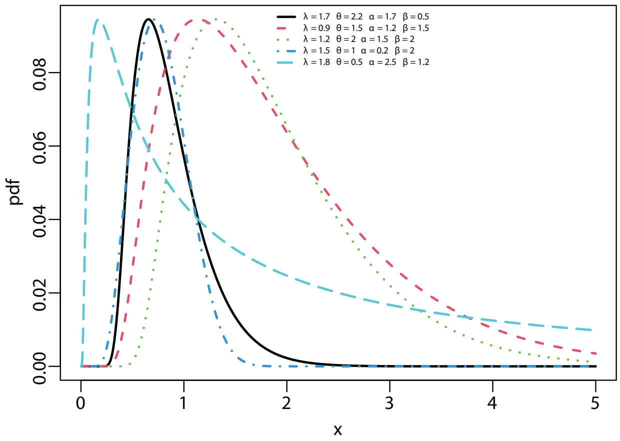


**Figure 1:** Pdf of NRQE-T2G (α,β,θ,λ).

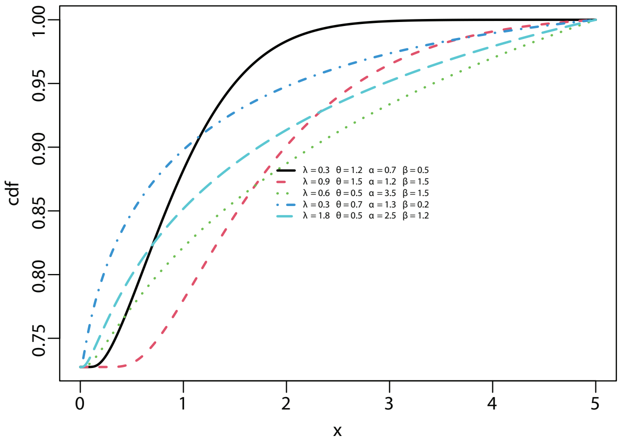


**Figure 2:** Cdf of NRQE-T2G (α,β,θ,λ)n

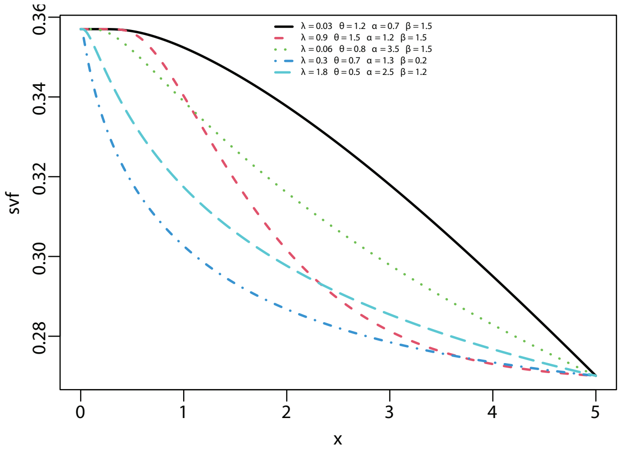


**Figure 3:** Survival function of NRQE-T2G (α,β,θ,λ)

## Properties of the NRQE-T2G

**The quartile function:** From the cumulative density function, we derived the quantile function by making x the subject of the formula which is given by:



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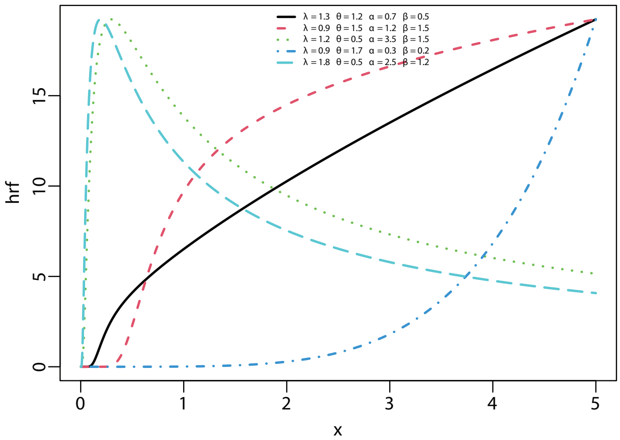


**Figure 4:** Hazard function of NRQE-T2G (α,β,θ,λ)

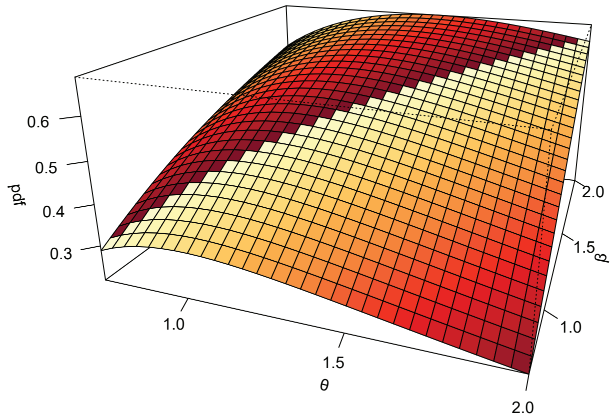


**Figure 5:** 3D pdf of NRQE-T2G (α,β,θ,λ)

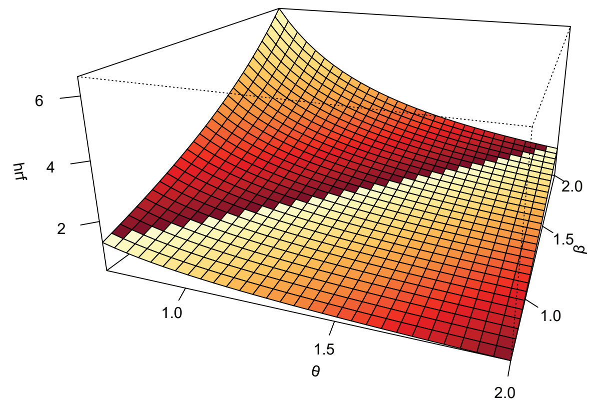


**Figure 6:** 3D hazard function of NRQE-T2G (α,β,θ,λ)

$$Q(u)=\left(-\frac{1}{\beta}\ln\left\{1-\left[1-\frac{\ln(1-u)}{\lambda}\right]\right\}^{-\alpha}\right)^{-\theta} \quad (14)$$

Where $u \in (0,1)$.

**The moments and it's associated measures:** In this section we talk about the Mean( $1^{st}$ Order Moment), Variance, $S^{th}$, $2^{nd}$, $3^{rd}$, $4^{h}$ Order Moment;



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The $s^{th}$ Moment

$$\mu_s' = \int_0^\infty x^s f(x)dx; \quad \mu_s' = \int_0^\infty x^s \left( \frac{\lambda\theta\beta x^{-\theta-1} e^{-\{\beta x^{-\theta}+\lambda[\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}}-1]\}}}{\alpha[\{1-e^{-\beta x^{-\theta}}\}^{\frac{1+\alpha}{\alpha}}]} \right) dx$$

$$\mu_s' = \frac{\lambda\theta\beta}{\alpha} \int_0^\infty \left( \frac{x^{s-\theta-1} e^{-\{\beta x^{-\theta}+\lambda[\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}}-1]\}}}{[\{1-e^{-\beta x^{-\theta}}\}^{\frac{1+\alpha}{\alpha}}]} \right) dx;$$

$$\mu_s' = \frac{\lambda\theta\beta}{\alpha} \int_0^\infty \left( x^{s-\theta-1} e^{-\{\beta x^{-\theta}+\lambda[\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}}-1]\}} [\{1-e^{-\beta x^{-\theta}}\}^{-\frac{1+\alpha}{\alpha}}] \right) dx$$

Since; $e^x = \sum_{j=0}^\infty \frac{x^j}{j!}$

Then;

$$e^{-\{\beta x^{-\theta}+\lambda[\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}}-1]\}} = \sum_{i=0}^\infty \frac{(-1)^i}{i!} \{\beta x^{-\theta} + \lambda[\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}} -1]\}$$

Let X~ NRQE-T2G (λ,α,β,θ), then the $S^{th}$ crude moment can be expressed as:

$$\sum_{i=0}^\infty \frac{(-1)^i}{i!} \{\beta x^{-\theta} + \lambda[\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}} -1]\}; \quad \sum_{i=0}^\infty \sum_{j=0}^i \frac{(-1)^i}{i!} \binom{i}{j} \beta^{i-j} x^{-\theta(i-j)} \lambda^j [\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}} -1]^j$$

$$\sum_{i=0}^\infty \sum_{j=0}^i \frac{(-1)^{i+j}}{i!} \binom{i}{j} \beta^{i-j} x^{-\theta(i-j)} \lambda^j [1-\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}}]^j;$$

$$\sum_{i=0}^\infty \sum_{j=0}^i \sum_{k=0}^\infty \frac{(-1)^{i+j+k}}{i!} \binom{i}{j} \binom{j}{k} \beta^{i-j} x^{-\theta(i-j)} \lambda^j [\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}}]^{\frac{-k}{\alpha}}$$

$$\sum_{i=0}^\infty \sum_{j=0}^i \sum_{k=0}^\infty \sum_{l=0}^\infty \frac{(-1)^{i+j+k+l}}{i!} \binom{i}{j} \binom{j}{k} \binom{\frac{k}{\alpha}+l-1}{l} \beta^{i-j} x^{-\theta(i-j)} \lambda^j \{e^{-\beta x^{-\theta}}\}^l$$

$$\sum_{i=0}^\infty \sum_{j=0}^i \sum_{k=0}^\infty \sum_{l=0}^\infty \frac{(-1)^{i+j+k+l}}{i!} \binom{i}{j} \binom{j}{k} \binom{\frac{k}{\alpha}+l-1}{l} \beta^{i-j} x^{-\theta(i-j)} \lambda^j e^{-l\beta x^{-\theta}}$$

Similarly;

$$\left(1-e^{-\beta x^{-\theta}}\right)^{-(\frac{1+\alpha}{\alpha})} = \sum_{m=0}^\infty (-1)^m \binom{\frac{1}{\alpha}+m}{m} e^{-m\beta x^{-\theta}}$$

$$\to \mu_s' = \frac{\lambda\theta\beta}{\alpha} \int_0^\infty \left( x^{s-\theta(i-j+1)} \sum_{i=0}^\infty \sum_{j=0}^i \sum_{k=0}^\infty \sum_{l=0}^\infty \sum_{m=0}^\infty \frac{(-1)^{i+j+k+l+m}}{i!} \binom{i}{j} \binom{j}{k} \binom{\frac{k}{\alpha}+l-1}{l} \binom{\frac{1}{\alpha}+m}{m} \beta^{i-j} \lambda^j e^{-\beta(l+m)x^{-\theta}} \right) dx$$

$$\mu_s' = \frac{\lambda\theta\beta}{\alpha} \sum_{i=0}^\infty \sum_{j=0}^i \sum_{k=0}^\infty \sum_{l=0}^\infty \sum_{m=0}^\infty \frac{(-1)^{i+j+k+l+m}}{i!} \binom{i}{j} \binom{j}{k} \binom{\frac{k}{\alpha}+l-1}{l} \binom{\frac{1}{\alpha}+m}{m} \beta^{i-j} \lambda^j \int_0^\infty \left( x^{s-\theta(i-j+1)} e^{-\beta(l+m)x^{-\theta}} \right) dx$$

Where

$$\phi = \sum_{i=0}^\infty \sum_{j=0}^i \sum_{k=0}^\infty \sum_{l=0}^\infty \sum_{m=0}^\infty \frac{(-1)^{i+j+k+l+m}}{i!} \binom{i}{j} \binom{j}{k} \binom{\frac{k}{\alpha}+l-1}{l} \binom{\frac{1}{\alpha}+m}{m} \beta^{i-j} \lambda^j$$



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Let $y = x^{-\theta}$ and transforming

$$\mu'_s = -\frac{\lambda\beta}{\alpha}\phi\int_0^\infty y^{-\left(\frac{s}{\theta}+i-j\right)}e^{-\beta(l+m)y}dy$$

Therefore the $S^{th}$ Moment is given by

$$\mu'_s = -\frac{\lambda\beta}{\alpha}\phi\frac{\Gamma\left(i-j-\frac{s}{\theta}+1\right)}{\{\beta(l+m)\}^{\left(i-j-\frac{s}{\theta}+1\right)}} \tag{15}$$

**The mean:** The first crude moment which is the mean is obtained by substituting $S$=1 in eq 15

$$\mu'_1 = -\frac{\lambda\beta}{\alpha}\phi\frac{\Gamma\left(i-j-\frac{1}{\theta}+1\right)}{\{\beta(l+m)\}^{\left(i-j-\frac{1}{\theta}+1\right)}} \tag{16}$$

The second, third, and fourth crude moments can be obtained by simply substituting $s$=2,3 and 4 in eq 15 respectively

$$\mu'_2 = -\frac{\lambda\beta}{\alpha}\phi\frac{\Gamma\left(i-j-\frac{2}{\theta}+1\right)}{\{\beta(l+m)\}^{\left(i-j-\frac{2}{\theta}+1\right)}}; \quad \mu'_3 = -\frac{\lambda\beta}{\alpha}\phi\frac{\Gamma\left(i-j-\frac{3}{\theta}+1\right)}{\{\beta(l+m)\}^{\left(i-j-\frac{3}{\theta}+1\right)}};$$

$$\mu'_4 = -\frac{\lambda\beta}{\alpha}\phi\frac{\Gamma\left(i-j-\frac{4}{\theta}+1\right)}{\{\beta(l+m)\}^{\left(i-j-\frac{4}{\theta}+1\right)}}$$

**The variance:** To obtain the variance, we can recall the basic statistical theorem that $E(X^2)-(E(X))^2$ ; where

$$E(X) = \mu'_1 = -\frac{\lambda\beta}{\alpha}\phi\frac{\Gamma\left(i-j-\frac{1}{\theta}+1\right)}{\{\beta(l+m)\}^{\left(i-j-\frac{1}{\theta}+1\right)}} \quad \text{and}$$

$$E(X^2) = \mu'_2 = -\frac{\lambda\beta}{\alpha}\phi\frac{\Gamma\left(i-j-\frac{2}{\theta}+1\right)}{\{\beta(l+m)\}^{\left(i-j-\frac{2}{\theta}+1\right)}}$$

Therefore

$$\sigma^2 = \mu'_2 - (\mu)^2 \tag{17}$$

**The Coefficient of variation (CV):** It is obtained by taking the square root of the variance of the NRQE-T2G distribution which gives the standard deviation and it's divided by the mean(first crude moment), therefore the c.v is as follows

$$C.V = \frac{\sigma}{\mu} = \frac{\sqrt{\mu'_2-(\mu)^2}}{-\frac{\lambda\beta}{\alpha}\phi\frac{\Gamma\left(i-j-\frac{1}{\theta}+1\right)}{\{\beta(l+m)\}^{\left(i-j-\frac{1}{\theta}+1\right)}}} \tag{18}$$

Where $\sigma = \sqrt{\mu'_2-(\mu)^2}$

**Skewness (SK):** In probability theory and statistics, skewness serves as an indicator of the asymmetry present in the probability distribution of a real-valued random variable concerning its mean. Skewness values can take on positive, zero, negative, or undefined characteristics. The skewness of the NRQE-T2G distribution is given as:



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If,

$$\varsigma_1=\left\{-\frac{\lambda\beta}{\alpha}\phi\frac{\Gamma\left(i-j-\frac{3}{\theta}+1\right)}{\{\beta(l+m)\}^{\left(i-j-\frac{3}{\theta}+1\right)}}\right\}+3(\mu_2'-(\mu)^2)\left\{\frac{\lambda\beta}{\alpha}\phi\frac{\Gamma\left(i-j-\frac{1}{\theta}+1\right)}{\{\beta(l+m)\}^{\left(i-j-\frac{1}{\theta}+1\right)}}\right\}$$

$$\varsigma_2=\left[\left\{-\frac{\lambda\beta}{\alpha}\phi\frac{\Gamma\left(i-j-\frac{1}{\theta}+1\right)}{\{\beta(l+m)\}^{\left(i-j-\frac{1}{\theta}+1\right)}}\right\}\right]^3 \qquad (19)$$

$$SK(X)=\frac{\varsigma_1-\varsigma_2}{[\mu_2'-(\mu)^2]^{1.5}}$$

**Kurtosis:** In the realm of probability theory and statistics, kurtosis, derived from the Greek words "kyrtos" or "kurtos," signifying "curved, arching," stands as a gauge for the "tailedness" of the probability distribution of a real-valued random variable. The kurtosis of the NRQE-T2G distribution is given by Figures 7,8.

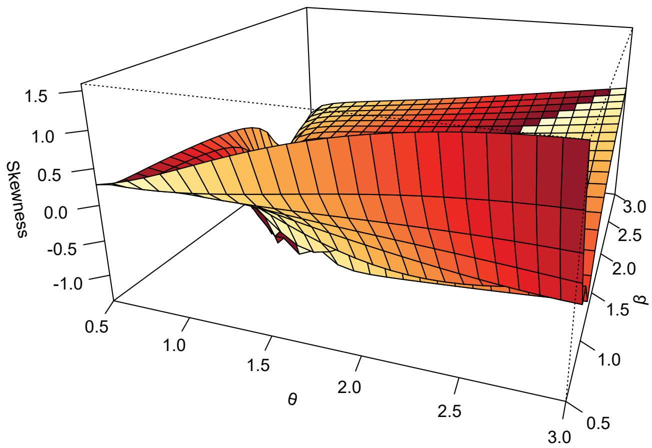


**Figure 7:** Skewness of NRQE-T2G (α,β,θ,λ)

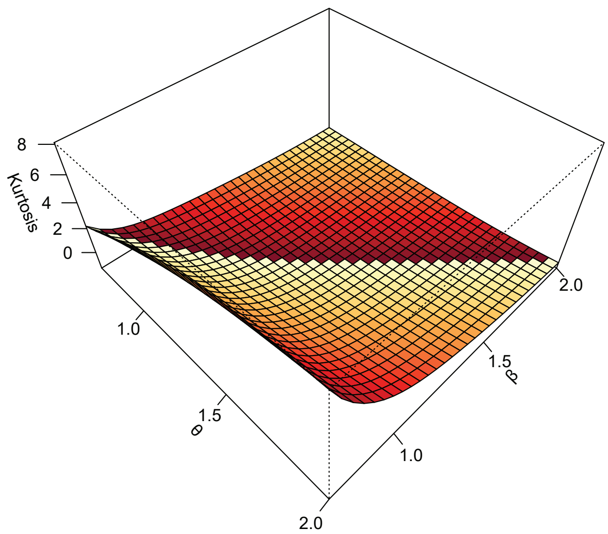


**Figure 8:** kurtosis of NRQE-T2G (α,β,θ,λ)n



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$$\varphi = \frac{\mu_4'}{\sigma^4}$$

$$\varphi = -\frac{\lambda\beta\phi\Gamma\left(i-j-\frac{4}{\theta}+1\right)}{\alpha\{\beta(l+m)\}^{\left(i-j-\frac{4}{\theta}+1\right)}[\mu_2'-(\mu)^2]^2} \qquad (20)$$

The 3D plot of the skewness in Figure 9 shows that the distribution is right-skewed but not heavy-tailed as the highest metric is a little above zero. Figure 10 is the 3D plot of the kurtosis illustrating a platykurtic scenario as the highest metric 2 which is less than 3 for Gaussian distribution.

**Index of sispersion:** The index of dispersion serves as a metric for measuring the dispersion of nominal variables and partially ordered nominal variables. It is commonly expressed as the ratio of the variance to the mean. The index of the dispersion of the NRQE-T2G distribution is given by

$$\gamma = \frac{\sigma^2}{\mu_1} = -\frac{[\mu_2'-(\mu)^2]\alpha}{\lambda\beta}\phi\frac{\{\beta(l+m)\}^{\left(i-j-\frac{1}{\theta}+1\right)}}{\Gamma\left(i-j-\frac{1}{\theta}+1\right)} \qquad (21)$$

## Order statistics

Order statistics is a crucial tool employed in addressing intricate issues within statistical methods. Let $X_1, X_2, X_3 \ldots, X_n$ be a random

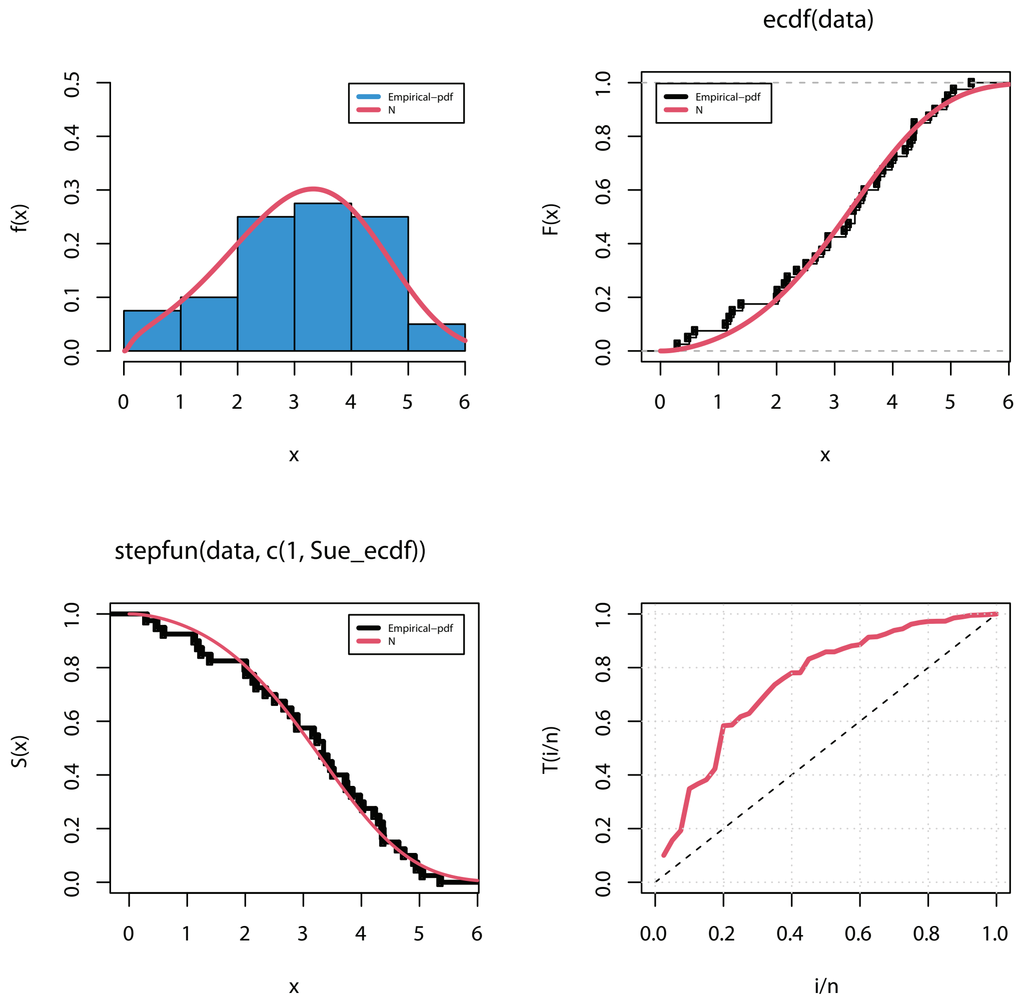


**Figure 9:** Density, cdf, survival and TTT plots for the data of 40 patients suffering from blood cancer (leukemia).



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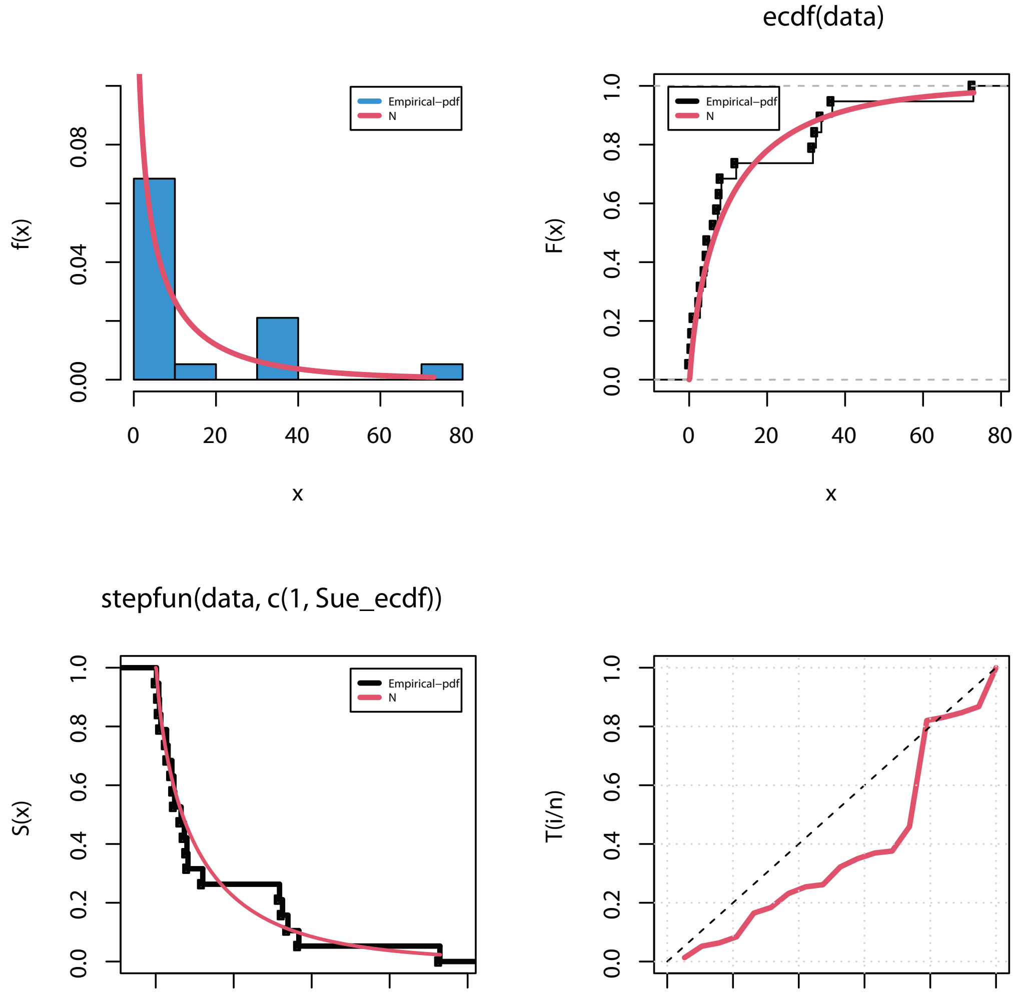


**Figure 10:** Density, cdf, survival, and TTT plots for the data on time to break down of an insulating fluid between electrodes at a voltage of 34 k.v. (minutes).

sample of $X_{(r)}$; (r = 1, 2, ..., n) and the $r^{(th)}$ order statistics can be obtained by arranging $X^{(r)}$ in ascending order of magnitude from the New Reduced Quantile Exponential - Type II Gumbel Distribution is

$$f_{r:n}(x;\alpha,\beta,\theta,\lambda)=\binom{n}{r}g(x)\{G(x)\}^{r-1}\{1-G(x)\}^{n-r}$$

$$f_{r:n}(x;\alpha,\beta,\theta,\lambda)=\binom{n}{r}\left(\frac{\lambda\theta\beta x^{-\theta-1}e^{-\{\beta x^{-\theta}+\lambda[\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}}-1]\}}}{\alpha[\{1-e^{-\beta x^{-\theta}}\}^{\frac{1+\alpha}{\alpha}}]}\right)\left[1-e^{-\lambda[\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}}-1]}\right]^{r-1}\left\{e^{-\lambda[\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}}-1]}\right\}^{n-r}$$

$$f_{r:n}(x;\alpha,\beta,\theta,\lambda)=\binom{n}{r}\left(\frac{\lambda\theta\beta x^{-\theta-1}e^{-\{\beta x^{-\theta}+\lambda[\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}}-1]\}+\lambda(n-r)[\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}}-1]}}{\alpha[\{1-e^{-\beta x^{-\theta}}\}^{\frac{1+\alpha}{\alpha}}]}\right)\left[1-e^{-\lambda[\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}}-1]}\right]^{r-1} \quad (22)$$

The pdf of the largest order statistics is obtained when r = n and that is;

$$f_{n:n}(x;\alpha,\beta,\theta,\lambda)=\left(\frac{\lambda\theta\beta x^{-\theta-1}e^{-\{\beta x^{-\theta}+\lambda[\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}}-1]\}}}{\alpha[\{1-e^{-\beta x^{-\theta}}\}^{\frac{1+\alpha}{\alpha}}]}\right)\left[1-e^{-\lambda[\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}}-1]}\right]^{n-1} \quad (23)$$

The pdf of the smallest order statistics is obtained when r = 1 and it is;



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$$f_{1:n}(x;\alpha,\beta,\theta,\lambda)=\left(\frac{n\lambda\theta\beta x^{-\theta-1}e^{-\{\beta x^{-\theta}+\lambda[\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}}-1]\}+\lambda(n-1)[\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}}-1]}}{\alpha[\{1-e^{-\beta x^{-\theta}}\}^{\frac{1+\alpha}{\alpha}}]}\right) \quad (24)$$

### Mean residual life function

The Mean Residual Life (MRL) function, also known as the remaining lifetime function or remaining life expectancy, is a concept used in reliability engineering, survival analysis, and actuarial science. It provides valuable insights into the remaining time until an event occurs, such as the failure of a system or the occurrence of a specific event.

The Mean Residual Life (MRL) function of the NRQE-T2G distribution is given by

$$m(x)=E[X-t\,|\,X>t]=\frac{1}{1-G(x)}\int_x^{\infty}[1-G(t)]dt \quad (25)$$

$$m(x)=\frac{\alpha}{\lambda\beta\theta x^{-(\theta+1)}\left\{1-e^{\beta x^{-\theta}}\right\}^{(-\frac{1+\alpha}{\alpha})}} \quad (26)$$

## Parameter estimation

Let $X_1,X_2,X_3,\ldots,X_n$ be a random sample of size n independently drawn from A New Reduced Quantile Exponential - Type II Gumbel Distribution with unknown parameters $\beta,\alpha,\lambda$ and $\theta$ which have been defined above. The likelihood function is defined as

$$L\left(x_i;\alpha,\beta,\theta,\lambda\right)=\Pi_{i=1}^{n}g\left(x_i;\alpha,\beta,\theta,\lambda\right)=$$
$$\frac{\lambda^n\theta^n\beta n}{\alpha^n}e^{-\sum_{i=1}^{n}\{\beta x^{-\theta}+\lambda[\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}}-1]\}}\Pi_{i=1}^{n}x^{-\theta-1}\Pi_{i=1}^{n}[\{1-e^{-\beta x^{-\theta}}\}^{-\frac{1+\alpha}{\alpha}}] \quad (27)$$

Where g(.) is defined in eq 10. The log-likelihood function becomes

$$logL\left(x_i;\alpha,\beta,\theta,\lambda\right)=n\log\lambda+n\log\theta+n\log\beta-\sum_{i=1}^{n}\{\beta x^{-\theta}+\lambda[\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}}-1]\}-(\theta+1)$$
$$\sum_{i=1}^{n}\log x-\left(\frac{1+\alpha}{\alpha}\right)\sum_{i=1}^{n}\log\{1-e^{-\beta x^{-\theta}}\} \quad (28)$$

Let $\varnothing=logL\left(x_i;\alpha,\beta,\theta,\lambda\right)$

Differentiating $\varnothing$ concerning $\beta,\alpha,\lambda$ and $\theta$ gives the following non-linear equation to be implemented in R using the **optim()** function

$$\frac{\partial\varnothing}{\partial\lambda}=\frac{n}{\lambda}-\sum_{i=1}^{n}\left(\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}}-1\right)$$

$\lambda$ has a closed-form solution given by:

$$\hat{\lambda}=\frac{n}{\sum_{i=1}^{n}\left(\{1-e^{-\beta x^{-\theta}}\}^{\frac{-1}{\alpha}}-1\right)}$$

$$\frac{\partial\varnothing}{\partial\theta}=\beta\sum_{i=1}^{n}x^{\theta}\ln x+\frac{\lambda\beta}{\alpha}\sum_{i=1}^{n}x^{\theta}\ln x\{1-e^{-\beta x^{-\theta}}\}e^{-\beta x^{-\theta}}$$



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$$\frac{\partial \varnothing}{\partial \beta}=\frac{n}{\beta}-\sum_{i=1}^{n}\left(x^{-\theta}-\frac{\lambda x^{-\theta}e^{-\beta x^{-\theta}}}{\alpha}\{1-e^{-\beta x^{-\theta}}\}^{-\left(\frac{1+\alpha}{\alpha}\right)}\right)$$

$$\frac{\partial \varnothing}{\partial \alpha}=\frac{\lambda}{\alpha^2}\sum_{i=1}^{n}\{1-e^{-\beta x^{-\theta}}\}^{-\left(\frac{1}{\alpha}\right)}\ln\{1-e^{-\beta x^{-\theta}}\}-\frac{n}{\alpha}$$

## Applications

The application of NRQE-T2G distribution to two different datasets is presented here and it's compared with the Burr Type XII by [19], New Exponentiated Weibull (NEX) by [20], Extended inverse exponential (EIE) by [21], Weibull Distribution by [22], Generalized inverted Exponential (GIE) by [23], Lomax distribution [18], Log-Normal Distribution studied by [24], Gamma distribution by [25] and [26], Kumaraswamy Weibull (KW) [27]. The first real-life data is on the Lifetimes (in days) of 40 patients suffering from blood cancer (leukemia) from one of the Ministry of Health Hospitals in Saudi Arabia was used by [28], also studied by [29] and [30]. The lifetime order (in years) is given in Table 1.

The measures of model performance for the distributions are the negative Log-Likelihood (NLL), Akaike Information Criterion (AIC), Corrected AIC (CAIC), Bayesian Information Criterion (BIC), Hannan– Quinn information criterion (HQIC), Cramer von Mises ($W^*$), Anderson Darling ($A^*$), while the Kolmogorov-Smirnov (K-S) statistic and the p-value determine the fitness of the distribution of the data.

Based on the results in Table 2, the proposed NRQE-T2G best fits the data of 40 patients suffering from blood cancer (leukemia) having the highest $p - value$ = 0.9625.

Table 3 contains the maximum likelihood estimates of the parameters of NRQE-T2G together with those of the competing distributions. The Lomax distribution parameter estimates are weird. The implication is that it cannot be used to fit the data on leukemia.

From Figure 9, the proposed NRQE-T2G model plots namely density, CDF, survival and TTT plots approximate the empirical lines. These suggest the fitness or adequacy of the model.

The second real-life data is on time to break off an insulating fluid between electrodes at a voltage of 34 k.v. (minutes). These data originated from [31], where Wayne Nelson presents the breakdown time of an insulating fluid between electrodes $34kV$. The times, in minutes, are as follows Table 4:

The measures of model performance for the distributions are the negative Log-Likelihood (NLL), Akaike Information Criterion (AIC), Corrected AIC (CAIC), Bayesian Information Criterion (BIC), Hannan– Quinn information criterion (HQIC), Cramer von Mises ($W^*$), Anderson Darling ($A^*$), while the Kolmogorov-Smirnov (K-S) statistic and the p-value determine the fitness of the distribution to the data.

Based on the results in Table 3, the proposed NRQE-T2G best fits the data of 40 patients suffering from blood cancer (leukemia) having the highest $p - value$ = 0.7986, Kumaraswamy Weibull (KW) Distribution didn't perform well due to the dataset and was replaced by Exponentiated Weibull (EW) Distribution Table 5.

Based on Table 6, the Weibull distribution did not give a close parameter estimate for the parameter β. Figures 10 shows that the proposed model also approximates the empirical results based on the density, CDF, survival, and TTT plots. The TTT plot is essentially a tool for determining whether a model improves a system or not. In particular, if the curve is concave, the model improves the system otherwise the system deteriorates. In the leukemia data, the system deteriorates while the system improves based on the time to break down of an insulating fluid data. The NRQE-T2G is better behaved with the time to break down of insulating fluid data since the theoretical plot of the model PDF in Figure 1 has the same shape as the density plot in Figure 10. There could be no better explanation for why the TTT plot of the leukemia data in Figure 9 is concave down depicting a decaying system.

**Table 1:** The data of 40 patients suffering from blood cancer (leukemia).

| 0.315 | 0.496 | 0.616 | 1.145 | 1.208 | 1.263 | 1.414 | 2.025 |
|---|---|---|---|---|---|---|---|
| 2.036 | 2.162 | 2.211 | 2.37 | 2.532 | 2.693 | 2.805 | 2.91 |
| 2.912 | 3.192 | 3.263 | 3.348 | 3.348 | 3.427 | 3.499 | 3.534 |
| 3.767 | 3.751 | 3.858 | 3.986 | 4.049 | 4.244 | 4.323 | 4.381 |
| 4.392 | 4.397 | 4.647 | 4.753 | 4.929 | 4.973 | 5.074 | 5.381 |



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**Table 2:** Analytical measures of performance and fitness using the data of 40 patients suffering from blood cancer (leukemia).

| Method | NLL | AIC | CAIC | BIC | HQIC | $A^*$ | $W^*$ | KS | p - value |
|---|---|---|---|---|---|---|---|---|---|
| NRQE-T2G | -67.35 | 141.6509 | 142.7938 | 148.4064 | 144.0935 | 0.3250 | 0.0460 | 0.0794 | 0.9625 |
| Burr XII | -92.86 | 189.7188 | 190.0431 | 193.0965 | 190.9401 | 3.7911 | 0.6770 | 0.3232 | 0.0005 |
| NEX | -71.34 | 146.6779 | 147.0022 | 150.0556 | 147.8991 | 1.0583 | 0.1672 | 0.1309 | 0.4994 |
| EIE | -79.68 | 163.3504 | 163.6747 | 166.7281 | 164.5717 | 2.4340 | 0.4130 | 0.2210 | 0.0403 |
| Weibull | -69.56 | 143.1195 | 143.4438 | 146.4972 | 144.3407 | 0.7797 | 0.1198 | 0.1181 | 0.6328 |
| Gamma | -73.55 | 151.1038 | 151.4282 | 154.4816 | 152.3251 | 1.4854 | 0.2415 | 0.1572 | 0.2764 |
| Lomax | -85.78 | 175.5563 | 175.8806 | 178.9341 | 176.7776 | 1.4964 | 0.2434 | 0.3002 | 0.0015 |
| GIE | -85.84 | 175.69 | 176.0143 | 179.0677 | 176.9113 | 3.3875 | 0.5972 | 0.2268 | 0.0327 |
| Log norm | -75.03 | 162.6563 | 162.9806 | 166.032 | 163.8776 | 2.3850 | 0.4054 | 0.1989 | 0.0846 |
| KW | -70.41 | 148.4583 | 149.6012 | 155.2139 | 150.9009 | 0.9397 | 0.1471 | 0.1515 | 0.3179 |

**Table 3**: The MLEs using the data of 40 patients suffering from blood cancer (leukemia).

| Method | MLEs | | | |
|---|---|---|---|---|
| | $\alpha$ | $\beta$ | $\lambda$ | $\theta$ |
| NRQE-T2G | 0.0424 | 3.1609 | 0.2842 | 0.3722 |
| Burr XII | - | 0.3077 | - | 3.0168 |
| NEX | 0.0183 | 2.7195 | - | - |
| EIE | 0.9979 | - | - | 2.2108 |
| Weibull | 2.4519 | 3.4905 | - | - |
| Gamma | 3.4155 | 0.9283 | - | - |
| Lomax | 19320525 | - | 60676478 | - |
| GIE | 2.6374 | - | 3.6035 | - |
| Log Norm | 1.0593 | 0.7029 | - | - |
| KW | 1.0043 | 0.0685 | 1.0841 | 2.0371 |

**Table 4**: The data on time to breakdown of an insulating fluid between electrodes at a voltage of 34 k.v. (minutes).

| 0.96 | 4.15 | 0.19 | 0.78 | 8.01 |
|---|---|---|---|---|
| 31.75 | 7.35 | 6.50 | 8.27 | 33.91 |
| 32.52 | 3.16 | 4.85 | 2.78 | 4.67 |
| 1.31 | 12.06 | 36.71 | 72.89 | |

**Table 5**: Analytical measures of performance and fitness using the data on time to break off an insulating fluid between electrodes at a voltage of 34 k.v. (minutes).

| Method | NLL | AIC | CAIC | BIC | HQIC | $A^*$ | $W^*$ | KS | p - value |
|---|---|---|---|---|---|---|---|---|---|
| NRQE-T2G | -67.91 | 143.6380 | 146.4952 | 147.4158 | 144.2774 | 0.3280 | 0.0529 | 0.1405 | 0.7986 |
| Burr XII | -71.45 | 146.8967 | 147.6467 | 148.7855 | 147.2163 | 0.6234 | 0.0986 | 0.2227 | 0.2614 |
| NEX | -68.19 | 140.3772 | 141.1272 | 142.2660 | 140.6968 | 0.3434 | 0.5591 | 0.1455 | 0.7641 |
| EIE | -69.58 | 143.1640 | 143.9140 | 145.0529 | 143.4837 | 0.4614 | 0.0817 | 0.2475 | 0.1643 |
| Weibull | -68.39 | 142.7253 | 143.4753 | 144.6142 | 143.0450 | 0.4035 | 0.0687 | 0.1952 | 0.4113 |
| Gamma | -68.62 | 161.7076 | 162.4576 | 163.5965 | 162.0273 | 0.4699 | 0.0826 | 0.4471 | 0.0006 |
| Lomax | -68.42 | 140.8468 | 141.5968 | 142.7357 | 141.1665 | 0.3094 | 0.0458 | 0.1479 | 0.7466 |
| GIE | -72.44 | 148.8721 | 149.6221 | 150.7610 | 149.1918 | 0.9310 | 0.1445 | 0.2297 | 0.2306 |
| Log norm | -68.41 | 140.8757 | 141.6257 | 142.7645 | 141.1953 | 0.2970 | 0.0424 | 0.1448 | 0.7689 |
| EW | -68.14 | 142.2833 | 143.8833 | 145.1166 | 142.7628 | 0.3013 | 0.0466 | 0.1425 | 0.7844 |



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Table 6: The MLEs use the data on time to break down an insulating fluid between electrodes at a voltage of 34 k.v. (minutes).

| Method | MLEs | | | |
|---|---|---|---|---|
| | $\alpha$ | $\beta$ | $\lambda$ | $\theta$ |
| NRQE-T2G | 0.7255 | 0.0063 | 0.0578 | 2.7956 |
| Burr XII | - | 0.2937 | - | 1.7381 |
| NEX | 0.0635 | 0.8878 | - | - |
| EIE | 0.0087 | - | - | 0.1244 |
| Weibull | 0.7002 | 7.7324 | - | - |
| Gamma | 1.9013 | 9.6646 | - | - |
| Lomax | 2.0326 | - | 16.7537 | - |
| GIE | 0.5215 | - | 1.1083 | - |
| Log Norm | 1.7036 | 1.4829 | - | - |
| EW | 2.8344 | 0.4598 | 2.6802 | - |

## Conclusion

So far, this article has presented a new Exponential-G generator of families of distributions by implementing a reduced quantile function of Lomax distribution on the known $T$-$X(Y)$ generator. A special submodel was considered by using the Type-II Gumbel distribution as a baseline. Its properties and estimation were studied. From the two real-life applications investigated, the TTT (total time on test) plots provide evidence indicating escalating failure rates. The model comparison tables show that the NRQE-T2G distribution gives a better fit to these data when compared to other contesting distributions. Therefore, the suggested NRQE-T2G distribution can be used to model data with decreasing, increasing, and reversed bath-tub failure rate shapes.

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