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Research Article

Lorentz Transformation and time dilatation

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Abstract

We consider two inertial frames S and S' and suppose that frame S' moves, for simplicity, in a single direction: the X -direction of frame S with a constant velocity v as measured in frame S.

Using homogeneity of space and time we derive a modified Lorentz Transformation (LT) between two inertial reference frames without using the second postulate of Einstein, i.e., we do not assume the invariant speed of light (in vacuum) under LT.

Roughly speaking we suppose: (H) Any clock which is at rest in its frame measures a small increment of time by some factor $s=s(v)$. As a corollary of relativity theory (H) holds with Lorentz factor $1/\gamma$. For $s=1$ we get the Galilean transformation of Newtonian physics, which assumes an absolute space and time. We also consider the relation between absolute space and Special Relativity Theory, thereafter STR.

It seems here that we need a physical explanation for (H).

We introduce Postulate 3. The two-way speed of light in X' and Z' -directions of the frame S' are c and outline derivation of (LT) in this setting. Note that Postulate 3 is a weaker assumption than Einstein's second postulate.

Introduction

The work is in progress and in this introductory paper, we will outline a mathematical model motivated by the special theory. We hope that work in this direction can contribute to a better understanding of Special Relativity Theory (STR).

The reader should have in mind that Physics is not Mathematics, and in Physics notions are not always defined rigorously as in Mathematics. Physics works by making mathematical models and judges them by how well they describe reality[1]. Mathematical models have an independent interest in mathematics and can help to clarify some phenomena in reality, and experimental verification of claims is important for physics [1].

Most physicists agree with Relativity Theory (RT) and some of them consider that it is the greatest triumph of the human mind.

Some scientists criticize the Foundations of the Relativity Theory, the lack of logical and physical grounding for fundamental concepts in the special and general relativity theory, such as time, space, the relativity of simultaneity, the second postulate, etc.

Bearing these criticisms in mind, we try to make a mathematical model motivated by the special theory of relativity (STR), in particular without using the second postulate of Einstein, and to consider it. The first principle of relativity (without the second) along with homogeneity of space and time and isotropy of space naturally gives rise to two possibilities, either we have Galilean transformation where space and time are absolute, or general Lorentz Transformation (LT) which includes Lorentz Transformation with a boost invariant speed c as the upper limit of all speeds. Most physicists believe that reality corresponds to LT, but pure mathematics from our assumption cannot decide that. Some promoters of the theory of relativity think probably in a metaphorical sense that Light has supernatural properties (and use the name ''god particle''); e.g. as far as I know for the sound the second postulate does not matter.

[1]According to legend, someone asked Einstein what is time, and he answered, "It's what a clock measures."

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Although most physicists accept the theory of relativity, there is a significant number of scientists who are critics of this theory [2-9]. In addition to the mentioned works, our research is also motivated by several works [10-18] that do not assume the second postulate. Bearing these papers in mind, we try to make a mathematical model motivated by the special theory of relativity (STR) which does not rely on the second postulate of Einstein. The purpose of our work is to clarify the situation from a mathematical point of view and develop a theory based on hypothesis (H), which we call time dilation and which is a substitute for the second postulate. As a corollary of our approach, we show that the two-way speed of light is invariant in all inertial frames.

First, we give a brief overview of the above-mentioned papers. Recall the usual derivation of (LT) (e.g., Einstein's original work, see also [5,16,19-24]) is based on the second postulate (the invariance of the speed of light). However, it turns out that the starting point can be (as is described, for example, in the second volume of the Course of Theoretical Physics by Landau and Lifshitz [25]), the assumption: that the influence that one particle exerts on another can not be transmitted instantaneously. Hence, some researchers conclude that there exists a theoretical maximal speed of information transmission that must be invariant, and it turns out that this speed coincides with the speed of light in a vacuum. Newton called the idea of action at a distance philosophically "absurd", and considered that gravity had to be transmitted instantaneously by some agent (the thing that takes an active role or produces a specified effect).

In a 1964 paper [17], Zeeman considered the causality-preserving property: A bijection, f , of space-time is said to be causal if, for all points x,y in space-time, $y-x$ is time-like and forward-pointing if and only if $f(y)-f(x)$ is also time-like and forward-pointing. This condition is weaker in a mathematical sense than the second postulate (the invariance of the speed of light) but it assures that the coordinate transformations are the Lorentz transformations. Goldstein obtained a similar result using inertiality (the preservation of time-like lines) [18].

Guerra and Abreu considered questions of absolute space and relativity [11]. In particular, they assume that there is one frame where the one-way speed of light in a vacuum is the same in all directions of space and equal to c. This frame can be identified with the rest frame, and it is shown that this frame is unique. They have denoted this rest frame as Einstein's frame.

They show that the meaning of the Principle of Relativity is not incompatible with the existence of a preferred, absolute, frame. Further, they establish that the one-way speed of light in a vacuum is not c in moving inertial frames (the two-way speed of light of course is) and simultaneity is absolute, contrary to what results in Einstein's relativity. The general expressions for the transformation of coordinates between inertial frames are obtained. Therefore we believe that it makes sense to consider models that modify the second postulate and that in the future there will be new theories that better approximate reality.

Our consideration is concerned with the abstract notion of time. We suppose time measuring devices at every point of a frame that read ''time'' as scalar which has properties of real numbers. In one part of the manuscript, we suppose homogeneity of space and time without Postulate 2, and in the other part we consider some result of STR's supposed Postulate 2.

In [13], Datta presents a new derivation of rotation-free Lorentz Transformation (LT) between two inertial reference frames without using the second postulate of Einstein, i.e., he does not assume the invariant speed of light (in vacuum) under LT. He finds a general transformation rule between two inertial frames where a speed, invariant under that transformation, arises naturally. This idea first came into light by a mathematician Ignatowski [14] around 1910. For additional literature on the subject, we refer to [13]. Therefore, the principle of relativity along with homogeneity of space and time and isotropy of space naturally gives rise to two possibilities, either we have Galilean transformation where space and time are absolute, or we have Lorentz Transformation with a boost invariant speed c as the upper limit of all speeds. According to Datta, most physicists believe (Nature works in accordance with LT) is the second possibility. It has interesting corollaries and it happens that c is the speed of an Electromagnetic wave in a vacuum. Our consideration is also based on the validity of velocity reciprocity, see [16]. Velocity reciprocity means that the velocity of an inertial frame S with respect to another inertial reference frame S' is the opposite of the velocity of S' with respect to S.

Definition and background

Reference frames and relative motion

Reference frames play a crucial role in relativity theory. The term reference frame as used here is an observational perspective in space that is not undergoing any change in motion (acceleration) (so, at rest or constant velocity), from which a position can be measured along 3 spatial axes say x,y,z. In addition, a reference frame has the ability to determine measurements of the time t of events using a "clock" (any reference device with uniform periodicity).

An event is an occurrence that can be assigned a single unique moment and location in space relative to a reference frame: it is a "point" in spacetime (x,y,z,t). Since the speed of light is constant in relativity irrespective of the reference frame, pulses of light can be used to unambiguously measure distances and refer back to the times that events occurred to the clock, even though light takes time to reach the clock after the event has transpired.

For example, the explosion of a firecracker may be considered to be an "event". We can completely specify an event by its four spacetime coordinates: The time of occurrence and its 3-dimensional spatial location define a reference point. Let's call this a reference frame S. An event is something that happens at a certain point in spacetime, or more generally, the point in spacetime itself. In any inertial frame, an event

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is specified by a time coordinate t or sometimes ct and a set of Cartesian coordinates x,y,z to specify position in space in that frame. We use notation a = (t,x,y,z) and $\mathbf{a}' = (t', x', y', z')$. Subscripts label individual events.

Standard configuration

To gain insight into how the spacetime coordinates measured by observers in different reference frames compare with each other, it is useful to work with a simplified setup with frames in a standard configuration. With care, this allows mathematical simplification without loss of generality in the conclusions that are reached. In Figure 1 below, two Galilean reference frames (i.e., conventional 3-space frames) are displayed in relative motion. Frame S belongs to a first observer O, and frame S' belongs to a second observer O'. The x,y,z axes of frame S are oriented parallel to the respective primed axes of frame S'. The frame S' moves, for simplicity, in a single direction: the x-direction of the frame S with a constant velocity v as measured in the frame S. The origins of frames S and S' are coincident when time t=0 for frame S and $t'=0$ for frame S'.

Since there is no absolute reference frame in relativity theory, the concept of "moving" doesn't strictly exist, as everything may be moving with respect to some other reference frame. Instead, any two frames that move at the same speed in the same direction are said to be comoving. Therefore, S and S' are not comoving.

By e we denote euclidean distance in R^3.

Einstein synchronisation (or Poincaré-Einstein synchronisation)

Throughout this paper we suppose that clocks in any inertial frames satisfy the hypothesis of continuity of time (HCT): For every $\epsilon > 0$ there is δ>0 such that if $e(A,B)<\delta$ and a light signal is sent at time τ_1 from clock 1 at A to B and it arrives at time τ_2 at B then $|\tau_2 - \tau_1| < \epsilon$.

Einstein synchronization (or Poincaré-Einstein synchronization) is a convention for synchronizing clocks in inertial frames at different places by means of signal exchanges. This synchronization method was used by telegraphers in the middle 19th century but was applied to light signals by Henri Poincaré and Albert Einstein, who recognized its fundamental role in relativity theory. Its principal value is for clocks within a single inertial frame.

Consider synchronisation in an inertial frame S. According

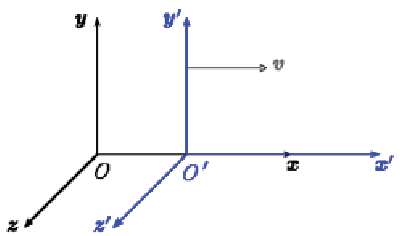

Figure 1:

to Albert Einstein's prescription from 1905, a light signal is sent at a time τ_1 from clock 1 at A to clock 2 at B and immediately back, e.g. by means of a mirror. Its arrival time back at clock 1 is τ_2. This synchronisation convention sets clock 2 so that the time τ_3 of signal reflection is defined to be

$$\tau_3 = \tau_1 + \tfrac{1}{2}(\tau_2 - \tau_1) = \tfrac{1}{2}(\tau_1 + \tau_2).$$

The same synchronisation is achieved by transporting a third clock from clock 1 to clock 2 "slowly" (that is, considering the limit as the transport velocity goes to zero). The literature discusses many other thought experiments for clock synchronisation giving the same result.

The problem is whether this synchronisation does succeed in assigning a time label to any event in a consistent way.

As far as we know most physicists do not accept attempts to negate the conventionality of this synchronisation.

Definition 1: *Consider an inertial frame S. Suppose that a light signal is sent at a time τ_1 from clock C_A at A to clock C_B at B and immediately back, e.g. by means of a mirror, and that its arrival time back at clock C_A is τ_2. If $\tau_2 - \tau_1 = 2e(A,B)/c$, where e denotes Euclidean distance, we say that clock C_A and C_B are in the average two-way synchronisation. If clocks in an inertial frame S satisfy the Hypothesis of Continuity of Time (HCT) and any two clocks in S are in the average two-way synchronisation, we say that clocks in S satisfy the two-way synchronisation condition.*

From a mathematical point of view, we think that this issue deserves further consideration. In particular let us direct the reader to the work Malament David B [26], where a symmetrical relation of causal connectability is used.

For our purposes the following definition is convenient

Definition 2: *Let S be reference frame with the origin O. We flash light from O at moment 0(when the clock there shows time 0) and when a flash of light reaches the clock at M, it begins running. In this setting, we say that clocks in S are synchronized in a standard way with respect to the origin.*

Question 1: *Suppose that frames S and S' are in a standard configuration and clocks in S are synchronized with respect to the clock C_o at the origin O (here we use that the speed of light is c in S between O and arbitrary point M). Let $t' = t'_x$ is time of clock $C_{O'}$ when O' is at position x in S and t=t_x is time of clock C_x. It seems that t' is defined without use Postulate 2 (i.e. that the speed of light is c in S'). Is $t = \gamma t'$ in this setting?*

Using notation in the above question, by Homogeneity of space and time, we find $t'_{x_2} - t'_{x_1} = \lambda(t_{x_2} - t_{x_1})$ and therefore $t' = \lambda t$. Let the rod R of length l' is at rest in S'. Then the length l of the rod R in S is independent of the position of the beginning point of the rod. If two rods of length l'_1 and l'_2 are at rest in S', then $l_2/l_1 = l'_2/l'_1$. Hence $l = al'$ and therefore $x = ax' + vt$. Note that in STR $a = \lambda = \gamma^{-1}$.

In general, we can ask:

Question 2: *Let S and S' be frames in standard configuration*

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with reference devices in S and S' with uniform periodicity which measure "time" and consider some signal which is independent of frames. Are there devices that measure the same speed of signal in both frames?

STR is based on two postulates:

Postulate 1: The laws of physics are invariant (identical) in all inertial frames of reference (that is, frames of reference with no acceleration).

Postulate 2: The speed of light in a vacuum is the same for all observers, regardless of the motion of the light source or observer.

Let $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ and $\gamma_{inv} = \sqrt{1-v^2/c^2}$. In STR the Lorentz transformation for frames in standard configuration can be shown to be (see for example [13,19,25,27,28] and literature cited there):

If an observer in S records an event t,x,y,z, then an observer in S records the same event with coordinates t',x',y',z' and we have

$$t' = \gamma\left(t - \frac{vx}{c^2}\right), \quad (2.1)$$

$$x' = \gamma(x - vt), \quad (2.2)$$

$$y' = y, \quad (2.3)$$

$$z' = z, \quad (2.4)$$

Where v is the relative velocity between frames in the x-direction, c is the speed of light, and

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

is the Lorentz factor.

The homogeneity of space and time

Through this paper (if we do not state otherwise) if we refer to two frames S and S' in relative motion v we suppose that:

(H_o): S and S' are frames in standard configuration.

In particular, it means that the frame S' moves, for simplicity, in a single direction: the positive x - direction of frame S with a constant velocity v as measured in frame S.

If in addition, we suppose Postulate 2, then we have LT:

$$x' = \gamma(x - vt), \quad (3.1)$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right). \quad (3.2)$$

We suppose here that the synchronisation is achieved in a frame S by transporting clocks from the origin which shows time zero when the clock C_0 shows time zero to the position x "slowly" (that is, considering the limit as the transport velocity goes to zero).

It is convenient to use notation and the setting described by: (A) Let x'_o be a point which is fixed at S' and time intervals between events $E_1 = (x_1,t_1)$ and $E_2 = (x_2, t_2)$ which coincide with $E'_1=(x'_0, t'_1)$ and $E'_2= (x'_0, t'_2)$.

Consider for a moment that (LT) holds. Next from (LT1), we find $x = x'/\gamma + vt$ and if we substitute in (LT2), we have $t' = t/\gamma - x'v/c^2$ and therefore

$t'_2 - t'_1 = (t_2 - t_1)/\gamma$

Motivated by this formula using the notation in the setting described by (A) we can consider a more general hypothesis:

t'_2-t'_1 $= \lambda(t_2 - t_1)$, where λ is independent of x'_0 .

Also motivated only by the homogeneity of space and time (without the second postulate) we can suppose that in the setting described by (A):

(H_1): t'_2-t'_1 $= \lambda(t_2 - t_1)$, for arbitrary x'_0 , t'_1,t'_2, where $\lambda = \lambda(v)$.

By symmetry from (H_1) it follows

t'_2- t'_1

Thus for frames in a relative motion (H_1) is equivalent to (H_2).

To compare the general transformation obtained from (H_2) it is convenient to introduce $\alpha = \lambda^{-1}$.

We call $\alpha = \lambda^{-1}$ the general coefficient of the time dilatation.

If in particular $\lambda = \gamma_{inv}$, then (H_1) has the form

(L) t_2'-t_1 $= \gamma_{inv}$(t_2-t_1) .

If (H_2) holds with $\alpha=\gamma$ we say that clocks in S and S' satisfy the Lorentz time dilatation condition (L-TD).

(L-TD) follows from the second postulate and it is a substitution for it. Namely, if (L-TD) holds, then the two-way speed of light measures along x-axis and x' -axis in S and S' respectively is c.

Various experiments confirmed both time dilation and the twin paradox, i.e. the hypothesis (L-TD). Bailey et al. (1977) [29] measured the lifetime of positive and negative muons sent around a loop in the CERN Muon storage ring. This experiment confirmed the hypothesis that clocks sent away and coming back to their initial position are slowed with respect to a resting clock (time dilation and the twin paradox). It is interesting that Albert Einstein and Max Born tried to explain the aging concerning the twin paradox as a direct effect of acceleration. But it turns out that neither general relativity nor even acceleration, are necessary to explain the effect.

Question 3: But we do not know whether there is a convincing explanation based on reality why clocks in motion are satisfying relation (L-TD).

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Symmetry in the physical sciences

We are going to use symmetry (as that term is understood in the physical sciences) in the next deduction, and note that this procedure is generally accepted in physics. As for rigor in the mathematical sense, see Question 4.

Now we consider that the hypothesis (H_2) holds and derive modified (LT).

Proposition 3.1: *Let S and S' be frames in standard configuration and suppose that S' moves in a single direction: the x-direction of frame S with a constant velocity v as measured in frame S and that (H_1) holds. Then* $x' = \lambda^{-1}(x - vt)$.

This can be considered as a generalization of LT.

Proof: We advise the reader to follow the proof using Figure 2. We can suppose that events $E=(x,t)$ and $E' = (x', t')$. $t' > 0$, coincide and fix for a moment $M=x$ and $M' = x'$. Let $x_0=vt$. Therefore there is a point x_0 such that events (x_0,t) and $(0,t_{0'})$ coincide. Set $M_0=x_0$.

To visualize imagine that the rod R is placed along $O'M'$ in S'.

The length of the rod R in S is (1) $l = x-x_0=x-vt$.

Next, by symmetry we can consider that S moves in a single direction: the x' -direction of frame S' with a constant velocity $-V$ as measured in frame S'. Thus we can imagine that M is moving to the origin O' in S' with velocity $-V$. Next suppose that when M arrives to coincide with the origin O' in S', the devices at S and S' show time t_1 and t'_1 respectively. Observer in S measures the length of the rod as $l=v(t_1-t)$ and observer in S' measures $l' = v(t'_1 - t')$. In this setting clock C_M which is at rest in S shows the increment of time t_1-t and by (H_2) $t_1 - t = \lambda(t'_1 - t')$ and therefore $l = v(t_1 - t) = v\lambda(t'_1 - t') = \lambda l'$. Thus by (1) $l= (x-vt)$ equals $x'\lambda$. Hence $x' = \lambda^{-1}(x - vt)$.

Question 4: *We used symmetry in the previous inference. It seems that in the mathematical sense, we need to give an additional explanation. Why we can consider that S is at rest and draw some conclusions, and then consider that they (S and S') can change roles and apply it to S'. For example, if S is a fixed system, it seems we cannot apply symmetry.*

In addition, suppose that the rod of length l' is placed at rest in S'.

Now suppose that we have time measuring devices at every point of any given frame and that two frames S and S' are in standard configuration.

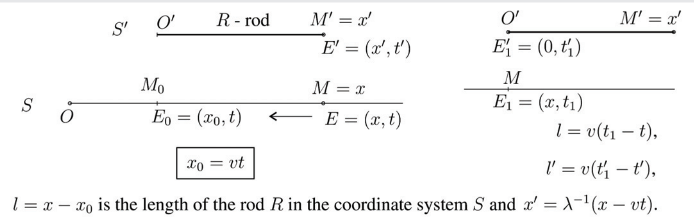

Figure 2:

Proposition 3.2: *If two frames S and S' are in standard configuration and (H_2) holds, then*

1. If the rod R of length l' is placed at rest in S'. Then an observer in S measures $l = \lambda l'$.

2. $x' = \lambda^{-1}(x - vt)$.

3. $t' = \lambda^{-1}(t - \frac{(1-\lambda^2)x}{v})$.

4. We can adjust time measuring devices in S and S' such that the speed of light is the same in S and S'.

5. If $\lambda = \gamma^{-1}$, then $(1-\lambda^2)x / v = vx / c^2$. Thus we get LT.

Outline of proof: We can suppose that the end of the rod is at O' at $t' = 0$ and $x' = -l' = A$ and that when A arrives at the origin O in S, the devices at S and S' show time t_1 and t'_1.

Then observers in S and S' measure that the length of the rod is $l = vt_1$ and $l' = vt'_1$ respectively. By (H_2) $t_1 = \lambda t'_1$ and therefore $l = vt_1 = v\lambda t'_1 = \lambda l'$. Next $(x - vt)$ equals $l = x'\lambda$. We can consider that S moves in a single direction: the x' -direction of frame S' with a constant velocity $-v$ as measured in frame S'. Hence by (1) $x' = \lambda^{-1}(x - vt)$. Here we can consider that x is at rest in S. Note here that 2) in Proposition 3.2 is stated as Proposition 3.1. [W]

It is suitable accordingly LT to introduce $\alpha = \lambda^{-1}$. By Proposition 3.1 $x' = \alpha(x - vt)$ and by symmetry $x = \alpha(x' + vt')$. Hence

$$t' = \alpha(t - \frac{(1-\alpha^{-2})x}{v}).$$

Lorentz time dilatation

In [25], Cuvaj Camillo gave a Survey of Langevin's Work in Relativity.

Langevin's light-clock

Suppose that we have two inertial frames S and S' in standard configuration with relative velocity v and consider point $M' = (0,0,L)$ in S'. We flash light signal from O' to M' and reflects back to O' in time interval t_A.

If the total round-trip time of the pulses in S is t_B using Pythagorean theorem and the second postulate we get $t_B = \gamma t_A$. Note that it follows directly from (L). Langevin, see for example [25], in order to explain time dilation visualize it by a thought experiment using a light-clock (originally developed by Lewis and Tolman in 1909.

Motivated by Langevin's approach we consider the following thought experiment: (B): Let's imagine straight lines long enough with rails (as track) and a train on rails moving at a speed v relative to the track. We can identify the frame xy-axis of S with track and frame S' with train. Next, we can imagine a box (Rectangular prism) Q on a train of perfectly reflecting walls wherein a light signal reflects back and forth

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from opposite faces(which is a light clock). Box Q is posted on the train with bases faces A and B and height L' such that

a) The horizontal box: the bases are parallel to yz-plane.

b) The vertical box: the bases are parallel to xy-plane.

We can imagine a "light clock" as a box of perfectly reflecting walls where a light signal successively reflects on opposite faces A and B.

It is interesting that Langevin considers the case a) and determines the Lorentz factor γ. Let C_o be a clock which is at rest on the base face A in S'. In case b) we can use a moving frame S' consisting of two mirrors at a distance L' and that speed of light measured along a line orthogonal on x,y axis is c and Pitagorian theorem. Let t, t' be the time of a light signal to move from A to A and reflects back to A in S and S' (measured by the clock $C_{o'}$) respectively. If we suppose that S is a fixed frame (we can not use symmetry here) and that the speed of light is c in z'-direction in S' then by Pitagorian theorem $c^2t^2 = c^2t'^2 + c^2v^2$ and hence $t' = t/\gamma$.

In case a) suppose that T, T' be time for a light signal to move from A to B and reflects back to A in S and S' (measured by the clock $C_{o'}$) respectively. Let L be height of the box measured in S. Then

$$T = \frac{2Lc}{c^2 - v^2}$$

and let $V = 2L'/T'$ be the two-way speed of light in S'. If $V = c$, then $T' = 2L'/c$ and using $T' = T/\gamma$ we find $L' = \gamma L$. We summarize:

Proposition 4.1: *Suppose the setting described by (B) above.*

1. In the case b) of the vertical box, if the speed of light is c in z'-direction in S' then $t' = t/\gamma$.

2. In the case a) of the horizontal box, $T = \frac{2Lc}{c^2 - v^2}$. In addition, if the two-way speed of light in S' in x' and z'-directions is c, then $T' = T/\gamma$ and $L' = \gamma L$.

Now we introduce a hypothesis that is a weaker assumption than Einstein's second postulate.

Postulate 3. The two-way speed of light in x' and z'-directions S' are c.

Using the above consideration we can derive.

Proposition 4.2: *Let S and S' be frames in standard configuration and suppose that S is the rest frame and that S' moves in a single direction: the x-direction of frame S with a constant velocity v as measured in frame S and that Postulate 3 holds. Then in the case a) of the setting (B), $T' = T/\gamma$ and $L' = \gamma L$.*

Proposition 4.3: *Let S and S' be frames in standard configuration and suppose that S' moves in a single direction: the x-direction of frame S with a constant velocity v as measured in frame S and that (L-TD) holds. Then the two-way speed of light measures along x-axis and x'-axis in S and S' respectively is c.*

Note that we do not suppose here that S or S' is fixed frame.

Outline of proof: Consider the setting (B) described by the case a). Then by (L-TD) we have $T' = T/\gamma$ and by symmetry we find $L' = \gamma L$. Hence

$$T = \frac{2Lc}{c^2 - v^2} = 2L\gamma^2/c$$

and

$$V = 2L'/T' = \frac{2\gamma L}{T/\gamma} = 2\gamma^2 L/T = c. \quad \square$$

Remark 1: *It seems that the proof works if we suppose only synchronization Hypothesis continuity of time (HCT) without some special synchronization. It seems that (L-TD) and (HCT) imply some kind of synchronization.*

Let's imagine straight lines long enough with rails (as track) and a train on rails moving at a speed v relative to the track and imagine a box (Rectangular prism) Q on a train of perfectly reflecting walls wherein a light signal reflects back and forth from opposite faces (which is a light-clock). Here we consider the track as S and the system on the train as S'.

In [20] Wu considers that Einstein's Velocity Time Dilation is nothing but an imagination or a pure mathematical time.

A corollary of SRT is that at any given moment the travelling clock in the frame S' is running slow in the ''stationary'' inertial frame S, but based on the relativity principle one could equally argue that S - clock is running slow in S' inertial frame. Roughly speaking clock which is at rest in its own frame is running slow. In relativity theory clock which is at rest in its own frame measures small the increment of time by the Lorentz factor $1/\gamma$. It seems here that we need a physical explanation for it.

Recall in Langevin's thought experiment a "light-clock" is used.

Question 5: *Whether a "light-clock" measures real physical time and whether an atomic clock, an ordinary clock measures the same time as a "light clock"?*

Suppose that K and S are in relative motion and (L-TD) holds.

In this situation, we can use symmetry.

If event $E_1 = (x,t)$ in S coincide with event $F_1 = (\xi,\tau)$ in K and when O arrives at ξ at moment t_1 in K the clock C_o at O shows time t_1 then $x=(t_1-t)v$. By symmetry, we can imagine that ξ moves to O and arrives there at events $F_2 = (\xi,\tau_1)$ which coincide with $E_2=(0,t_1)$. By (L-TD) $t_1-t= \gamma\,(\tau_1-\tau)$ and therefore

(i) $x = \gamma(\xi - v\tau)$.

Absolute frame and absolute time

If we suppose that K is an absolute frame and that clocks in K are synchronized in a standard way with respect to origin

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(as in Definition 2), then the above-defined time in K can be considered as absolute time. In the frame K we denote the origin by O^* and use coordinates (ξ, η, ζ, τ).

Proposition 4.4: *Let K and S be two inertial frames in standard configuration with relative velocity V. Suppose that K is an absolute frame. We have two possibilities:*

1. The time defined in K is considered as absolute time.

2. We can synchronize clocks in S with respect to the origin O.

In the case (1) $x = \xi - v\tau$.

If in addition, we suppose Postulate 3 in the case (2) we get $x = \gamma(\xi - v\tau)$.

Outline of proof: Recall that we suppose that clocks in K are synchronized wrt the origin O^* as in Definition 2. In the case (1), we suppose that clocks in K define an absolute time. For example if point x in S coincide with ξ (with events $(\xi,\tau,)$) we use the clock $C\tau$) to measure time in S (or that the clock C_x at x shows time $t=\tau$).

If event (x,t) in S coincides with event (ξ,τ) in K and when O arrives at ξ at moment τ_1 then we have

(i) $x = (\tau_1 - \tau)v = \xi - v\tau$.

In case (2) we have STR with absolute frame.

We can not use symmetry between S and K, but using Proposition 4.2 we derive (ii). [*W*]

Question 6: *Suppose that frames S and S' are in a standard configuration and clocks in S are synchronized with respect to the clock C_o at the origin O (here we use that the speed of light is c in S between O and arbitrary point M). Let $t' = t'_x$ is time of clock $C_{O'}$ when O' is at position x in S and t=t_x is time of clock C_x. It seems that t' is defined without the use of Postulate 2 (ie. that the speed of light is c in S'). In this setting whether $t = \gamma t'$?*

We plan to add further results in a forthcoming paper. In particular, we plan to consider the rest frame and use other hypotheses related to (*L-TD*) (and Postulate 2) and connect STR with hyperbolic geometry, the geometry of Minkowski space, and Möbius transformations.

Conclusion

The work is in progress and in this introductory paper, we have outlined a mathematical model motivated by the special theory. We hope that work in this direction will contribute to a better understanding of Special Relativity Theory (STR).

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