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Research Article

Study of vibration shock processes of non-linear mechanical systems with distributed parameters

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Abstract

In practice, under the conditions of perfection and constructive development of modern equipment and machines, nonlinear mechanical systems with distributed parameters are often encountered, which, depending on the principles of operation, are affected by vibration shock. Therefore, the study of vibration shock processes of the mentioned systems has great theoretical and practical importance and as a result to determine the optimal parameters of vibration protection devices to ensure their safe operation. In our case, the displacement field of two interacting non-linear mechanical systems with distributed parameters is considered, when their interaction is of vibration shock nature. Obviously, the mentioned events are more pronounced when the self-oscillation frequency of one or both systems momentarily approaches the frequency of forced vibration shock processes. In addition, critical moments are fixed during the phase shifts of forced oscillations of oscillatory systems, in this case, the frequencies of forced oscillations approach mutually opposing phase moments. By choosing the optimal parameters of hysteresis losses, it is possible to almost exclude sub-harmonic modes superimposed on the main resonance modes in vibration shock processes.

During hysteresis losses of the parabolic type, the value of μ changes automatically in connection with impulsive loads, which will allow us to transfer the vibration shock processes to automatic modes and, accordingly, the practically safe operation of the mentioned systems.

Introduction

In our case, the displacement field of two interacting distributed parameter non-linear mechanical systems is considered, when their interaction is vibration-shock in nature, and there is a pre-interval $\Delta > 0$ between them (Δ is the interval between the displacements of non-linear mechanical systems), and the interaction angle changes in the interval $0 < \alpha < 90^o$, and the vibration shock process has a periodic character. In this case, the interaction of the mentioned systems is described by the function

$$f_j\left(x_j,t\right)=f_j\left(x_j,t+T\right),\ \ j=1,2$$

where T is the interaction period; x_1 and x_2 – are the coordinates of the longitudinal movement of the system, and either $x_j\in\left[\ell_{1j},\ell_{2j}\right]$ (where ℓ_{1j} and ℓ_{2j} are the corresponding geometric dimensions of the system). Obviously, the vibration displacement of both systems has a random character and is completely described by the function $U_j\left(x_j,t\right)$ during vibration shock processes, the equations of motion of the interacting systems in our case will have the form (j=1, 2)

$$\left[\frac{\partial^2}{\partial t^2}-a_j^2\frac{\partial^2}{\partial x_j^2}-2b_j\frac{\partial^2}{\partial t\,\partial x_j}\right]U_j\left(x_j,t\right)=$$
$$=\delta_{1j}\cdot f_0\sin\left(\omega t+\phi\right)\cdot\Phi_1\left(x_2,\dot{x}_2\right)\delta\left(x_2-x_{22}\right)- \qquad (1)$$
$$-\left[\left(-1\right)^j\Phi_2\left(U,\dot{U}\right)\sin\alpha+m_j\frac{\partial^2U_j(x_j,t)}{\partial t^2}\right]\delta\left(x_j-\ell_j\right),$$

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where δ is the Dirac functionaries; $\delta_{1,j}$ – Corner symbol; a_j – propagation speed of elastic waves in systems; b_j – viscous resistance coefficient of vibration shock systems [1];

m_j – mass of interacting systems; $f_0 \sin(\omega t + \phi)$ – disturbing power; x_{22} – the distance from the point of concern to the anti-vibration means; $t \in [0,T]$, $U(t) = U_2(\ell_{12},t) - U_1(\ell_{11},t)$ – relative coordinate; $\Phi_1(x_2,\dot{x}_2)$ – rate of damping (harmful energy absorption) in the first system; $\Phi_2(U,\dot{U})$ – power function transferred to the second system. These functions characterize the vibration shock force between the systems. In this case, the initial and boundary conditions for finding the displacement field of interacting systems are zero. The action of gravity is not taken into account in the equations of motion. Vibration shock occurs during the rotation of the first system about its oZ axis, with angular velocity ω, the first system is a completely rigid body [2]. The mechanical model of interacting systems is presented in the first figure, where $\vec{v}$ the velocity vector field is determined by the following relationship Figure 1.

$$\vec{v} = [\vec{\omega}, \vec{r}] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{K} \\ 0 & 0 & \omega \\ x & y & z \end{vmatrix} = -\omega y \vec{i} + \omega x \vec{j}$$

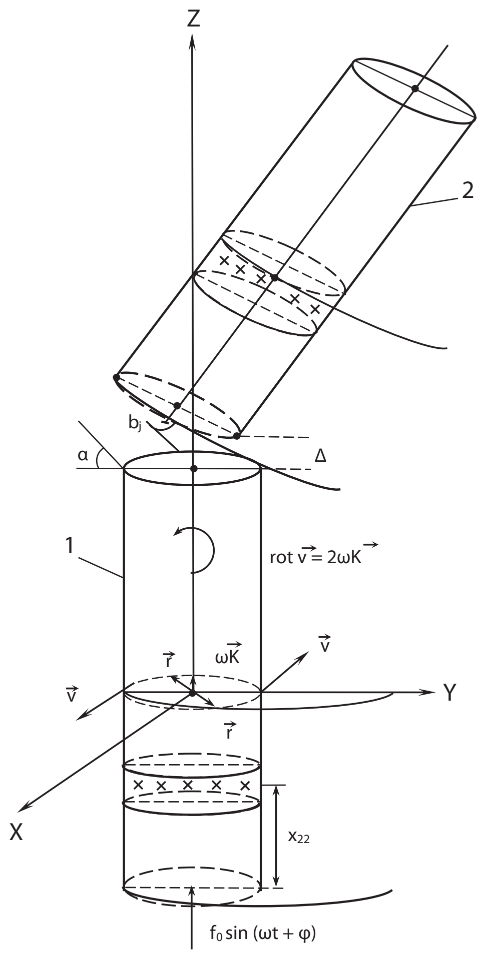

Figure 1: Model of two interacting mechanical systems.

$\vec{v}$ – the rotor of the speed vector field characterizes the system winding process, which in our case is determined by the following formula

$$rot\,\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{K} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\omega y & \omega x & 0 \end{vmatrix} = \vec{i}\,0 + \vec{j}\,0 + 2\omega \vec{K}$$

as we can see, in this case, $rot\,\vec{v}$ is a constant vector and is directed in the direction of the OZ axis, and its modulus is equal to double the angular velocity of rotation $|rot\,\vec{v}| = 2\omega$.

From the first equation, let's move on to integration-differential equations of interacting systems, where $\Phi_1(x_2,\dot{x}_2)$ and $\Phi_1(U,\dot{U})$ and are non-linear functions, and we are looking for a solution considering the periodicity of T. Periodic modes that satisfy the first equation will also satisfy the following equations

$$U(x_j,t) = U_{oj}(x_j,t) - (-1)^j \int_0^T \int_{\ell_{1j}}^{\ell_{2j}} \mathrm{v}_j(x_j,y_j,t-\tau) \times \\ \times \sin\alpha\, \Phi_2(U,\dot{U})\,\delta(y_j - \ell_{j1})\,dy_j\,d\tau, \quad (2)$$

where the periodic Green's function for nonlinear mechanical systems with distributed parameters has the form

$$\mathrm{v}_j(x_j,y_j,t) = T^{-1} \cdot L_{Kj}(x_j,y_j)\exp(iK\omega t),$$

$$T = 2\pi\omega^{-1};$$

$$L_{Kj}(x_j,y_j) = L_{Kj}(x_j,y_j,iK\omega).$$

The last function characterizes the dynamic operation of the given system with superimposed frequency $k\omega$ from point y_j to point x_j.

The function $U_{0j}(x_j,t)$ in this case is defined by the following equation

$$U_{0j}(x_j,t) = \int_0^T \int_{\ell_{1j}}^{\ell_{2j}} \mathrm{v}_j(x_j,y_j,t-\tau)\delta_{1j} \cdot f_0 \sin(\omega t + \varphi) \times \\ \times \delta(y_j - x_2)\Phi_1(y_j,\dot{y}_j)\,d\,y_j \cdot d\tau. \quad (3)$$

It can be seen from equation (2) that the displacements defined in one period of vibration shock during interaction take the form of a parametric representation

$$U_j(x_j,t) = U_{0j}(x_j,t) - (-1)^j F_0\,\mathrm{v}_j(x_j,\ell_{1j},t-\varphi), \quad (4)$$

where the momentum of the force and the phase of the oscillations satisfy the conditions $F_0 = m(1+R)\dot{U}_-(\varphi) \geq 0;\ U(\varphi) = \Delta$

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, U is the relative coordinate; $R[0,1]$ – system state recovery coefficient. Accordingly, we will have $F_0 = F_0^- + F_0^+$ – which is the sum of the loading and unloading stresses $F_0^- = n\dot{U}_-$; $F_0^+ = m\dot{U}_{t+}$, $m = \frac{m_1 m_2}{m_1 + m_2}$ – is the reduced mass [3].

The goal of our research is to determine $U_{0j}(x_j,t)$ and $v_j(x_j, \ell_{1j}, t+\varphi)$, and from equation (4) determine the field of corresponding displacements $U_j(x_j,t)$ and $U_j(x_j,0)$.

If we look for a general solution in the form of an expansion of the own forms of free oscillations, then we can present the periodic Green's function in the following form,

$$v_j(x_j,\ell_j,t) = \sum_{K=1}^{\infty} \frac{2C_{Kj}\sin\left(\ell_j^{-1}C_{Kj}x_j\right)\sin C_{Kj}\,\ell^{-\Gamma_{Kj}\omega_{Kj}t}}{\ell_j d_{Kj}\left(2C_{Kj}+\sin 2C_{Kj}\right)} \times$$
$$\times \frac{\sin\alpha\left[\sin\left(d_{Kj}t\right)+\sin d_{Kj}(T-t)\,\ell^{-\Gamma_{Kj}\omega_{Kj}t}\right]}{1+\ell^{-\Gamma_{Kj}\omega_{Kj}T}-2\ell^{-\Gamma_{Kj}\omega_{Kj}T}\cdot\cos\left(d_{Kj}T\right)} \qquad (5)$$

where $t\in[0,T]$, C_{Kj} are the roots of the following equation

$$C_j\,tg\,C_j = \ell_j \cdot m_j^{-1},$$
$$\Gamma_{Kj}\omega_{Kj} = a_j C_{Kj}\ell_j^{-2};\quad d_{Kj} = \omega_{Kj}\sqrt{1-\Gamma_{Kj}^2},$$

the ω_{Kj} are frequencies of the given system's own oscillations, in addition

$$\omega_{Kj}^2 = b_j C_{Kj}^2\ell_j^{-2},\quad \Gamma_{Kj}^2 < 1 \text{ and}$$

$$\Gamma_{Kj} = \mu\omega_{Kj}^2\left(4\pi\omega_j\right)^{-1};$$

μ is the coefficient of absorption of harmful energies, ω_j – are the frequencies of forced oscillations of vibration shock processes,

when $\omega_j = \omega_{Kj}$, then $v_j(x_j,\ell_j,t)\to\infty$.

Methods

In this case, an elastic-damping ring is included in both interacting systems, which is characterized by parabolic type hysteresis absorption ability, so all members of the equation (5) contain a multiplier with exponential suppression of oscillations, therefore, over time, the values of the second terms in the equation (4) approach zero, and the impulsive forced oscillations are defined only with the first members. To find the general solution $U_{oj}(x_j,t)$, in equation (3) it is allowed that $U_{oj} = 0$, as a result of which we get

$$U_{02}(x_1,t) = \int_0^T\int_0^{\ell_2} v_1(x_1,y,t-\tau)\Phi_1(y,\dot{y})f_0 \times$$
$$\times\sin(\omega\tau+\phi)\delta(y-x_{22})\,dy\,d\tau, \qquad (6)$$

where the hysteresis losses of harmful energies are described by the equation [4],

$$\Phi_1(y,\dot{y}) = K_2 y^2(1+\lambda\,\text{sgn}\,\dot{y}),$$

$$|\lambda|<1, \text{if } \dot{y}<0, \text{ then } \dot{U}_->0 \qquad (7)$$

K_2 – is the average dynamic stiffness of the anti-vibration agent; λ – is the rate of absorption of harmful energies of forced vibration shock oscillations [5]. Accordingly, from equation (5), we have

$$v_1(x_1,y,t-\tau) = \sum_{K=1}^{\infty}\frac{2C_{K1}\sin\left(y^{-1}C_{K1}\right)\sin C_{K1}\,\ell^{-\Gamma_{1j}\omega_{K1j}(t-\tau)}}{\ell_j d_{Kj}\left(2C_{Kj}+\sin 2C_{Kj}\right)}\times$$
$$\times\frac{\sin\alpha\left[\sin\left(d_{K1}\tau\right)+\sin d_{K1}(t-\tau)\,\ell^{-\Gamma_{K1}\omega_{K1}\tau}\right]}{1+\ell^{-2\Gamma_{K1}\omega_{K1}t}-2\ell^{-\Gamma_1\omega_{K1}t}\cdot\cos\left(d_{K1}-t\right)}. \qquad (8)$$

If we insert the equations (7) and (8) into the equation (6) and integrate, we get

$$U_{02}(x_1,t) = f_0\sin(\omega t+\varphi')\cdot\left|A_1(x_1,x_{22},i\omega)\right|\sin\alpha,$$

where φ' is a phase shift [6],

$$\varphi^1 = \varphi + \arg A_1;\ -\pi \le \arg A_1 \le \pi,$$

$\arg A_1$ is the principal value of the A_1 amplitude argument, and the modulus is defined by the equations [7]

$$\left|A_1(x_1,x_{22},i\omega)\right| = 2K_2(1\pm\lambda)x_{22}^2\sum_{K=1}^{\infty}\frac{C_{K1}}{\ell_2 d_{K1}}\times$$
$$\times\frac{\sin\left(C_{K1}x_1\right)\ell^{-\Gamma_{K1}\omega_{K1}t}\cdot\sin\left(C_{K1},x_{22}^{-1}\right)}{\left(2C_{K1}+\sin 2C_{K1}\right)\left[1+\ell^{-2\Gamma_{K1}\omega_{K1}t}-2\ell^{-\Gamma_{K1}\omega_{K1}t}-\cos d_{K1}t\right]}\times$$
$$\times\left[\frac{\ell^{-\Gamma_{K1}\omega_{K1}T}\left(\Gamma_{K1}\omega_{K1}\sin d_{K1}T-d_{K1}\cos d_{K1}T-\Gamma_{K1}\omega_{K1}\right.}{\omega_{K1}^2}-\frac{\cos d_{K1}T}{\Gamma_{K1}\omega_{K1}}\right], \qquad (9)$$

$$d_{K1} = \frac{\omega_{K1}\sqrt{4\pi^2\omega^2-\omega_{K1}^2\mu^2}}{4\pi\omega}.$$

The analysis of equations (4), (5), and (9) shows that even in the case of increased hysteresis losses of harmful energies in interacting vibration shock systems [8], it is impossible to completely exclude the impulsive loads acting on the systems and the corresponding critical and resonance events, which are

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accompanied by periodic vibration shock processes. Obviously, the mentioned events are more pronounced when the self-oscillation frequency of one or both systems momentarily approaches the frequency of forced vibration shock processes. In addition, critical moments are fixed during the phase shifts of forced oscillations of oscillatory systems, in this case, the frequencies of forced oscillations approach mutually opposing phase moments. It can be seen from equation (9) that by increasing the damping capacity, the amplitudes of both the current and resonant modes decrease, accordingly, the impulse loads acting on the systems decrease. It is not excluded that sub-harmonic resonance modes may also develop in systems during forced vibration shock processes. In this case, the work of dissipative forces in one period of vibration shock is determined by the display

$$E_D = \int_0^{\frac{2\pi \ell_2}{\omega}} K_2 y^2 \left(1 + \lambda \operatorname{sgn}\frac{\partial y}{\partial t}\right) dt , \tag{10}$$

and the work of the disturbing force takes into account the orthogonal of the harmonic function

$$E_B = \int_0^{\frac{2\pi \ell_2}{\omega}} f_0 \sin(\omega t + \varphi) \cdot \dot{y}(t)\, dt = \pi f_0 y \cdot \ell_2 \sin\varphi_0 , \tag{11}$$

where $\varphi_0 = \varphi + \varphi_2$, ($\varphi_2$ - is the new phase shift indicator). The equation for balancing the work of dissipative and disturbing forces will take the form in this case

$$\pi f_0 y \ell_2 \sin\phi_0 = E_D + \frac{(1-R) I^2}{2(1+R)} . \tag{12}$$

It can be seen from equation (12) that by selecting the optimal parameters of hysteresis losses,

it is possible to almost exclude sub-harmonic modes superimposed on the main resonant modes in vibration shock processes. The optimal values of impulses of forced oscillations, their duration and phases, which ensure minimum vibration loads of vibration shock systems, are determined by the following formulas

$$tg\varphi \le \frac{2\Gamma_{K1}\omega_{K1}\omega}{\omega_{K1}^2 - \omega^2};$$

$$F_0\omega\left|A_1\left(x_j, x_{22}, i\omega\right)\right| \le 4C_{K1} \sin\frac{d_{K1}x_j}{\ell_{1j}} \sin\frac{d_{K1}x_{22}}{\ell_{2j}} \times$$

$$\times\left[\ell_{1j}\left(2b_j + \sin 2b_j\right)\sqrt{\left(\omega_{K1}^2 - \omega^2\right)^2 + 4\Gamma_{K1}^2\omega_{K1}^2\omega^2}\right]^{-1}, \tag{13}$$

where U_{01}^{kr} - is the critical value of displacement when $\omega_{K1} = 4\pi\omega \cdot \mu^{-1}$, then $A_1 \to \infty$, i.e. $U_{02}(x_1, t)$ will take its maximum value when $U_{01} = 0$, then $U_1(x_1, t) = F_0 v_1(x_1, \ell_{1j}, t - \phi)$ and under the conditions mentioned above it will take its maximum value.

Conclusion

From the analysis of the inequalities (13) it can be seen that, in the case of parabolic type hysteresis losses, the value of μ changes automatically in relation to impulsive loads, which will allow us to transfer vibration processes to automatic modes and, accordingly, practically safe operation of the mentioned systems. The obtained results are of particular importance in heavy machinery construction, and their consideration and implementation will increase the safe and long-lasting operation of manufactured products.

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