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Research Article

The modern mathematical models in economics and finance

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Abstract

With this article, we open a new section in this journal: the application of mathematical methods in economics and finance. A few topics we would like to discuss to get started are corporate finance, investments, business valuation, taxation, and ratings. We describe shortly mathematical models in these areas. In the field of corporate finance, we discuss the foundations of two main theories of capital structure, the Modigliani-Miller and the modern theory of Brusov-Filatova-Orekhova (BFO theory). We compare them and describe the differences between them and their results. In the field of investments, we describe two modern investment models: (1) with debt repayment at the end of the project and (2) with uniform debt repayment and discuss their properties and applications. In business valuation, we discuss the problems that exist in this area and ways to solve them. In rating methodology, a new approach is devoted to the rating of non–financial issuers, as well as to long–term and arbitrary duration project rating. The key factors of a new approach are the adequate use of discounting of financial flows virtually not used in existing rating methodologies, and the incorporation of rating parameters (financial "ratios") into the modern theory of capital structure (Brusov–Filatova–Orekhova (BFO) theory). This article is devoted to the analysis of the theoretical mathematical methods and models based on first principles. The novelty of this consideration is due to the fact that we are considering and discussing recently developed mathematical models in economics and finance.

Introduction

Finance is a quantitative science, so the application of mathematical methods in finance is very important, as in economics. Although qualitative methods in economy and finance (expert valuations, surveys, etc.) are important, only quantitative methods can ensure adequate management decisions.

One of the most important advantages of quantitative methods is their high degree of objectivity and independence from the personality of the appraiser.

Among the quantitative methods, one can distinguish both purely computational and theoretical ones, which develop the main provisions on the basis of first principles. This article is devoted to the analysis of such basic mathematical methods and models. The novelty of this consideration is due to the fact that we are considering and discussing recently developed mathematical models in economics and finance.

In the field of corporate finance, we discuss the foundations of two main theories of capital structure: Modigliani-Miller [1-3] and the modern theory of Brusov-Filatova-Orekhova (BFO theory) [4-8]. We compare them and describe the differences between them and their results. In the field of investments, we describe two modern investment models: (1) with debt repayment at the end of the project and (2) with uniform debt repayment and discuss their properties and applications. In business valuation, we discuss the problems that exist in

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this area and ways to solve them. In rating methodology, a new approach is devoted to the rating of non–financial issuers, as well as to long–term and arbitrary duration project rating. The key factors of a new approach are the adequate use of discounting of financial flows virtually not used in existing rating methodologies, and the incorporation of rating parameters (financial "ratios") into the modern theory of capital structure (Brusov–Filatova–Orekhova (BFO) theory). This article is devoted to the analysis of the theoretical mathematical methods and models based on first principles. The novelty of this consideration is due to the fact that we are considering and discussing recently developed mathematical models in economics and finance.

A detailed comparison of the BFO theory with the MM theory and its numerous modifications is carried out in this review [9].

One of the further directions of investigation is the accounting of business risk along with financial risk. For this the incorporation of CAPM (Capital Asset Pricing Model) [10-12] and Fama–French models [13-15] into two major theories of capital structure – Brusov–Filatova–Orekhova (BFO) theory and Modigliani–Miller (MM) theory will be done. Some aspects of business risk accounting and the theory of capital structure are considered in [16-34].

Corporate finance

The problem of the capital cost and the capital structure, the influence of the capital structure on its cost, and the company capitalization are one of the main problems of corporate finance. Even the question of the existence of an optimal capital structure of companies (at which the company's value is maximum and the weighted average cost of capital is minimum) is open. Many theories and models, including the first quantitative theory of Nobel laureates Modigliani and Miller (MM) [1-3] do not solve the problem, and through a large set of restrictions (like the MM theory, for example) have little connection with the real economy. At the same time, qualitative theories and models based on an empirical approach do not allow for the necessary assessment.

Brusov, Filatova, and Orekhova [4-6] laid the foundations for modern corporate finance, investment, taxation, and ratings. It is based on the author's work on modifying the theory of capital cost and capital structure by Nobel laureates Modigliani and Miller, which led to the actual replacement of this theory by the modern theory by Brusov–Filatova–Orekhova (the BFO theory).

The authors departed from the Modigliani–Miller assumption about the eternity (infinity of lifetime) of companies and developed a quantitative theory for assessing the main financial indicators of companies of arbitrary age.

The results of modern BFO theory are very different from those of the Modigliani–Miller theory. They indicate that the latter, due to its perpetual nature, understates (often significantly) the weighted average cost of capital and equity of the company and significantly overstates (also often significantly) the company's valuation.

Such an incorrect valuation of key financial indicators of companies led to an underestimation of the associated risks and serious difficulties in making adequate management decisions. This was one of the implicit causes of the 2008 global financial crisis.

Within the framework of the BFO theory, a lot of qualitatively new effects were discovered in corporate finance, including an anomalous dependence of the cost of equity on leverage, which significantly changes the principles of the company's dividend policy; the effect of the "golden age" of the company [7,8] and many others.

The authors explicitly took into account inflation both in the Modigliani–Miller theory and in the Brusov–Filatova–Orekhova theory, with the help of which they discovered its non–trivial effect on the dependence of the cost of equity on leverage.

The established BFO theory allows the conduct of a valid valuation of the financial indicators of companies, such as the cost of raising capital, company value, etc. It allows the management of a company to make adequate decisions, which improves the effectiveness of the company management.

More generally, the introduction of a new system for assessing the financial performance of companies in financial reporting systems (IFRS, GAAP, etc.) will reduce the risk of a global financial crisis.

Within the framework of the BFO theory, an analysis of the well-known trade-off theory was done. It is shown that the assumption of risky debt financing (and a rising lending rate on the eve of bankruptcy) does not lead to an increase in the WACC, which continues to decrease with increasing leverage. This means there is no minimum for WACC versus leverage, and no maximum for company value versus leverage. This means that there is no optimal capital structure in the well-known trade-off theory, which proves its failure.

Recently two main theories of the capital structure – Brusov–Filatova–Orekhova and Modigliani–Miller – have been adapted to the established financial practice of the functioning of companies, taking into account the real conditions of their work (see [9] and references there). This made it possible to investigate the impact of frequent income tax payments p with advance income tax payments and payments at the end of reporting periods, as well as the impact of the company's variable income on its main financial results. In [9], an analysis of all existing theories of capital structure (with their advantages and disadvantages) was carried out in order to understand all aspects of the problem and make the right management decisions in practice. The role of the capital structure lies in the fact that the correct determination of the optimal capital structure allows the company's management to maximize the capitalization of the company and fulfill the long–term goal of the functioning of any company. In [9], the state of the theory of the structure of capital and the cost of capital is considered from the middle of the last century, when

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the first quantitative theory was created, to the present. The two main theories of Modigliani–Miller (MM) and Brusov–Filatova–Orekhova (BFO) are discussed and analyzed, as well as their numerous modifications and generalizations. The Brusov–Filatova–Orekhova (BFO) theory, its methodology, and its results are widely known [7]. Many authors use the BFO theory in practice.

Theoretical basis

The following notation will be used in the text below.

k_d, $w_d = \frac{D}{D+S}$ - the debt capital cost and debt capital share, $k_e, w_e = \frac{S}{D+S}$ - the equity capital cost and the equity capital share, and $L = D/S$ - the value of financial leverage, D - the debt capital value, S - the equity capital value, k0 - the equity capital cost at zero leverage level, g - income growth rate, p - frequency of tax on income payments, $WACC$ - the weighted average cost of capital, t - tax on profit, n - company age.

The Brusov–Filatova–Orekhova (BFO) theory and its perpetual limit– the Modigliani–Miller theory have recently been generalized to the established practice of the functioning of companies. This generalization took into account the real operating conditions of companies, such as variable income, frequent income tax payments, advance income tax payments, etc. This made it possible to investigate the impact of these conditions on its main financial performance [9]. The generalized Brusov–Filatova–Orekhova (BFO) theory as well as the generalized Modigliani – Miller theory allow the study of the influence of the conditions of the real functioning of companies on the dependence of the cost of equity on debt financing, as well as on the anomalous effect: on its existence and management [6].

Below we give a summary of the WACC formulas for BFO–theory as well as for MM–theory [9].

The classical BFO equation for $WACC$

$$\frac{1-(1+WACC)^{-n}}{WACC-g} = \frac{1-(1+k_0)^{-n}}{(k_0-g)\cdot\left(1-w_d t\left[1-(1+k_d)^{-n}\right]\right)} \qquad (1)$$

and the limit for perpetuity companies (MM limit)

$$WACC = k_0\cdot(1-w_d t) \qquad (2)$$

The formula for the equity cost comes from the definition of WACC

$$WACC = k_e w_e + k_d w_d (1-t), \qquad (3)$$

accounting that

$$w_e = \frac{1}{1+L}; w_d == \frac{L}{1+L} \qquad (4)$$

and is as follows

$$k_e = WACC(1+L) - Lk_d(1-t) \qquad (5)$$

The WACC formulas for BFO–theory and for MM–theory under the conditions of the real functioning of companies are presented below [9].

Variable income case

Income tax payments at the ends of periods

$$BFO: \frac{1-\left(\frac{1+g}{1+WACC}\right)^n}{WACC-g} = \frac{1-\left(\frac{1+g}{1+k_0}\right)^n}{(k_0-g)\cdot\left(1-w_d t\left[1-(1+k_d)^{-n}\right]\right)} \qquad (6)$$

$$MM: WACC = (k_0-g)\cdot(1-w_d t)+g \qquad (7)$$

Advance income tax payments

$$BFO: \frac{1-\left(\frac{1+g}{1+WACC}\right)^n}{WACC-g} = \frac{1-\left(\frac{1+g}{1+k_0}\right)^n}{(k_0-g)\cdot\left(1-w_d t\left[1-(1+k_d)^{-n}\right]\cdot(1+k_d)\right)} \qquad (8)$$

$$MM: WACC = (k_0-g)\cdot\left(1-w_d t\cdot(1+k_d)\right)+g \qquad (9)$$

Frequent income tax payments

Income tax payments at the ends of periods

$$BFO: \frac{1-(1+WACC)^{-n}}{WACC} = \frac{1-(1+k_0)^{-n}}{k_0\cdot\left(1-\frac{k_d w_d t}{p}\frac{\left[1-(1+k_d)^{-n}\right]}{(1+k_d)^{1/p}-1}\right)} \qquad (10)$$

$$MM: WACC = k_0\cdot\left(1-\frac{k_d w_d t}{p\cdot\left[(1+k_d)^{1/p}-1\right]}\right) \qquad (11)$$

Advance income tax payments

$$BFO: \frac{1-(1+WACC)^{-n}}{WACC} = \frac{1-(1+k_0)^{-n}}{k_0\cdot\left(1-\frac{k_d w_d t}{p}\frac{\left[1-(1+k_d)^{-n}\right]\cdot(1+k_d)^{1/p}}{(1+k_d)^{1/p}-1}\right)} \qquad (12)$$

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$$MM: WACC = k_0 \cdot \left(1 - \frac{k_d w_d t \cdot (1+k_d)^{1/p}}{p \cdot \left[(1+k_d)^{1/p} - 1 \right]} \right) \quad (13)$$

Simultaneous accounting of variable income in case of frequent income tax payments

Income tax payments at the ends of periods:

$$BFO: \frac{1 - \left(\frac{1+g}{1+WACC} \right)^n}{WACC - g} = \frac{1 - \left(\frac{1+g}{1+k_0} \right)^n}{(k_0 - g) \cdot \left(1 - \frac{k_d w_d t}{p} \frac{\left[1 - (1+k_d)^{-n} \right]}{(1+k_d)^{1/p} - 1} \right)} \quad (14)$$

$$MM: WACC - g = (k_0 - g) \cdot \left(1 - \frac{k_d w_d t}{p \cdot (1+k_d)^{1/p} - 1} \right) \quad (15)$$

Advance income tax payments

$$BFO: \frac{1 - \left(\frac{1+g}{1+WACC} \right)^n}{WACC - g} = \frac{1 - \left(\frac{1+g}{1+k_0} \right)^n}{(k_0 - g) \cdot \left(1 - \frac{k_d w_d t}{p} \frac{\left[1 - (1+k_d)^{-n} \right] \cdot (1+k_d)^{1/p}}{\left[(1+k_d)^{1/p} - 1 \right]} \right)} \quad (16)$$

$$MM: WACC - g = (k_0 - g) \cdot \left(1 - \frac{k_d w_d t \cdot (1+k_d)^{1/p}}{p \cdot (1+k_d)^{1/p} - 1} \right) \quad (17)$$

The general formula for equity cost, k_e, is as follows

$$k_e = WACC(1+L) - Lk_d(1-t). \quad (18)$$

To study the dependence of the cost of equity capital, ke, on various variables, it is first necessary to find the value of WACC and substitute it into the formula (18).

Modern investment Models

Investment models with debt repayment at the end of the project: The effectiveness of the investment project could be considered from two points of view: owners of equity and debt capital and only owners of equity. NPV in each case is calculated by two different methods: with and without separation of investment and debt flows. In the first case, discounting is done at two rates, and in the second case, two flows are discounted at one rate equal to WACC.

In the first case, the negative flows (debt and interest paid by equity holders) are equal to the (positive) flows received by debt holders and thus returned to the project. The only effect of leverage, in this case, is the effect of the tax shield created by tax incentives: the interest on the debt is included in the cost and thus reduces the tax base. For each period, the post-tax capital flow is

$$NOI(1-t) + k_d Dt \quad (19)$$

and at the initial time moment, T = 0 the value of investments is equal to $-I = -S - D$.

NOI, here is the net operating income (before taxes).

Investments at the time moment T = 0 are equal to $-S$ in the second case, and the capital flow per period is equal

$$(NOI - k_d D)(1-t). \quad (20)$$

We suppose that interests on debt are paid in equal shares of $k_d D$ during all periods. The principal repayment is made at the end of the last period.

The effectiveness of the investment project from the point of view of owners of equity

With flows separation: In this case, the expression for NPV has a view

$$NPV = -S + \sum_{i=1}^{n} \frac{NOI(1-t)}{(1+k_e)^i} + \sum_{i=1}^{n} \frac{-k_d D(1-t)}{(1+k_d)^i} - \frac{D}{(1+k_d)^n}$$
$$= -S + \frac{NOI(1-t)}{k_e} \left(1 - \frac{1}{(1+k_e)^n} \right) - D(1-t) \left(1 - \frac{1}{(1+k_d)^n} \right) - \frac{D}{(1+k_d)^n}. \quad (21)$$

The last term is the discounted (present) cost of the loan repaid in a single payment at the end of the last period n.

Without flows separation: In this case, operating and financial flows are not separated and are discounted, using the general rate, equal to WACC. NPV takes the following form:

$$NPV = -S + \sum_{i=1}^{n} \frac{NOI(1-t) - k_d D(1-t)}{(1+WACC)^i} - \frac{D}{(1+WACC)^n}$$
$$= -S + \frac{NOI(1-t) - k_d D(1-t)}{WACC} \left(1 - \frac{1}{(1+WACC)^n} \right) - \frac{D}{(1+WACC)^n}. \quad (22)$$

Investment models with uniform debt repayment

Above, we described investment models with debt repayment at the end of the project, which have proven themselves well in the analysis of real investment projects. In practice, however, the scheme of equal debt repayment throughout the project is more common.

As in the case of debt repayment at the end of the project, the effectiveness of the investment project is from two points of view: owners of equity and debt capital and only owners of equity. In the first case, the negative flows (debt and interest paid by equity holders) are equal to the (positive) flows received by debt holders and thus returned to the project. The only effect of leverage, in this case, is the effect of the tax shield created by tax incentives: the interest on the debt is included in the cost

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and thus reduces the tax base. NPV in each of these cases is calculated in two ways: with the separation of investment and credit flows and discounting payments at two different rates and without such separation when both flows are discounted at the same rate equal to WACC.

The Effectiveness of the investment project from the equity holders' points of view

With flows separation: In this case, the expression for NPV has a view

$$\mathrm{NPV} = -S + \sum_{i=1}^{n} \frac{\mathrm{NOI}(1-t)}{(1+k_e)^i} + \sum_{i=1}^{n} \frac{-k_d D \dfrac{n+1-i}{n}(1-t) - \dfrac{D}{n}}{(1+k_d)^i}$$

$$= -S + \frac{\mathrm{NOI}(1-t)\left(1-(1+k_e)^{-n}\right)}{k_e}$$

$$-\left(\frac{D}{n} + k_d D \frac{n+1}{n}(1-t)\right)\frac{1-(1+k_d)^{-n}}{k_d}$$

$$+ k_d \frac{D}{n}(1-t)\left\{\frac{(1+k_d)\left[1-(1+k_d)^{-n}\right]}{k_d^2} - \frac{n}{k_d(1+k_d)^n}\right\} \quad (23)$$

In perpetuity limit (let us call it Modigliani–Miller limit), one has

$$\mathrm{NPV} = -S + \frac{\mathrm{NOI}(1-t)}{k_e} - D(1-t).$$

Without flows separation: In this case, operating and financial flows are not separated and are discounted, using the general rate (as which, WACC can be selected).

The main debt repayment, which occurs evenly (by equal parts) at the end of each period, can be discounted either at the same rate WACC or at the debt cost rate k_d. Now we choose a uniform rate and the first option.

We still consider the effectiveness of the investment project from the perspective of the equity holders only.

$$\mathrm{NPV} = -S + \sum_{i=1}^{n} \frac{\mathrm{NOI}(1-t) - k_d D \dfrac{n+1-i}{n}(1-t) - \dfrac{D}{n}}{(1+\mathrm{WACC})^i}$$

$$= -S + \frac{\mathrm{NOI}(1-t) - \dfrac{D}{n} - k_d D \dfrac{n+1}{n}(1-t)}{\mathrm{WACC}} \cdot \left(1 - \frac{1}{(1+\mathrm{WACC})^n}\right)$$

$$+ \frac{k_d D}{n}(1-t)\left\{\frac{(1+\mathrm{WACC})\left[1-(1+\mathrm{WACC})^{-n}\right]}{\mathrm{WACC}^2} - \frac{n}{\mathrm{WACC}(1+\mathrm{WACC})^n}\right\} \quad (24)$$

In perpetuity limit (Modigliani–Miller limit) (turning to the limit $n \to \infty$ in the relevant equations), we have

$$\mathrm{NPV} = -S + \frac{\mathrm{NOI}(1-t) - k_d D(1-t)}{\mathrm{WACC}}.$$

The effectiveness of the investment project from the owners of equity and debt points of view

With flows separation: For consideration from the points of view of the owners of equity and debt, NPV takes a following form

$$\mathrm{NPV} = -I + \sum_{i=1}^{n} \frac{\mathrm{NOI}(1-t)}{(1+k_e)^i} + \sum_{i=1}^{n} \frac{k_d D \dfrac{n+1-i}{n} t}{(1+k_d)^i}$$

$$= -I + \frac{\mathrm{NOI}(1-t)\left(1-(1+k_e)^{-n}\right)}{k_e}$$

$$+ D\frac{n+1}{n} t \cdot \left[1-(1+k_d)^{-n}\right]$$

$$- k_d \frac{D}{n} t\left\{\frac{(1+k_d)\left[1-(1+k_d)^{-n}\right]}{k_d^2} - \frac{n}{k_d(1+k_d)^n}\right\} \quad (25)$$

Without flows separation: Without flows separation, the NPV has the following form

$$\mathrm{NPV} = -I + \sum_{i=1}^{n} \frac{\mathrm{NOI}(1-t) + k_d D \dfrac{n+1-i}{n} t}{(1+\mathrm{WACC})^i}$$

$$= -I + \frac{\mathrm{NOI}(1-t) + k_d D \dfrac{n+1}{n} t}{\mathrm{WACC}}\left(1 - \frac{1}{(1+\mathrm{WACC})^n}\right)$$

$$-\frac{k_d D}{n} t\left\{\frac{(1+\mathrm{WACC})\left[1-(1+\mathrm{WACC})^{-n}\right]}{\mathrm{WACC}^2} - \frac{n}{\mathrm{WACC}(1+\mathrm{WACC})^n}\right\} \quad (26)$$

Rating methodologies

Existing rating methodologies have a lot of shortcomings. One of the major flaws of all of them is a failure or a very narrow use of discounting. But even in those rare cases where it is used, it is not quite correct, since the discount rate when discounting financial flows is chosen incorrectly. In [35] a new approach to rating methodology is suggested, devoted to the rating of non–financial issuers, as well as to long–term and arbitrary duration project rating. The key factors of a new approach are 1) The adequate use of discounting of financial

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flows virtually not used in existing rating methodologies, 2) The incorporation of rating parameters (financial "ratios") into the modern theory of capital structure (Brusov–Filatova–Orekhova (BFO) theory). This on the one hand allows the use of the powerful tools of this theory in the rating, and on the other hand, it ensures the correct discount rates when discounting financial flows. We discuss also the interplay between rating ratios and leverage level which can be quite important in rating. All these create a new base for rating methodologies. The new approach to ratings and rating methodologies allows issuing of more correct ratings of issuers, making the rating methodologies more understandable and transparent.

Business valuation

The management by finance is the management of financial flows. We consider the role of the correct determination of discount rate in preventing abuse in business valuation, in investments, in determining the fair value of dividend income of shareholders, etc. In business valuation, unscrupulous appraisers manipulate the value of the discount rate for the raider capture of enterprises. In investments, an incorrect assessment of the discount rate leads to an incorrect assessment of the effectiveness of an investment project and does not allow ranking investment projects in order to select the most effective projects in conditions of limited investment resources (companies, municipal, and state). This can lead to misappropriation of public funds, including funds from national projects. The rights of shareholders to receive adequate profits can be violated when the company's management conducts an incorrect and ineffective dividend policy, due to the inability of the management to determine the correct amount of dividends (the economically justified amount of which is the equity cost), or with the deliberate violation of shareholders' rights in this area. Only the most modern versions of the Brusov–Filatova–Orekhova (BFO) theory, taking into account the conditions of the real functioning of the companies, allow you to correctly assess the discount rate and make an adequate management decision. One of the further directions of investigation is the accounting of business risk along with financial risk. For this the incorporation of CAPM (Capital Asset Pricing Model) [10-12] and Fama–French models [13-15] into two major theories of capital structure – Brusov–Filatova–Orekhova (BFO) theory and Modigliani–Miller (MM) theory will be done. Some aspects of business risk accounting and the theory of capital structure are considered in [16-34].

Conclusion

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