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## Review Article

# Excess Energy Released Due To Proton-Proton Collisions in Condensed Matter Could Be Due To Laboratory-Made Micro Black Holes

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## Abstract

While sodium metal dissolution in concentrated (2M) Epsom solution is very peaceful under stirring conditions in about 5 to 10s, its dissolution in a dilute (0.85) Epsom solution has turned very violent. Nearly 30s after the addition of Na, the solution exploded accompanied by the vaporization of the glass beaker. Only sodium aerosol and tiny molten needles of glass were seen around indicating a very high temperature (>1000 °C) has indeed been reached. The timing of the explosion has indicated that hydrogen released during sodium dissolution has got trapped in the salt solution and subsequent energy build-up has caused the excess energy release. A viable trapping mechanism involving the exchange of 2 hydrogen ions ( $H_2^{2+}$ ) with an  $Mg^{2+}$  ion in cavitation-induced nanocrystals of Epsom has been proposed in this regard. The two hydrogen ions are stabilized at the cation vacancy (cavity) by dative bonds with the two hydrogen ions in the water molecule trapped at a neighboring sulphate vacancy. Spring-like action between the two protons trapped at the cation vacancy due to oscillations caused by two opposing forces – one by cavitation and the other by coulombic repulsion eventually appear to have led to the collision of hydrogen ions at relativistic speeds in condensed matter. To start with, the two protons in the  $H_2^{2+}$  species are confined in a small space (72 pm cavity) at a single crystal cation lattice site in  $Na_2SO_4$  nanocrystal which increases their head-on collision probability at high energies. There is no upper limit to the energies achieved through p-p collisions in this species as the Cavitation-Coulombic Repulsion Oscillations (CCRO) can continue until the endpoint is reached. Regeneration of a nanocrystal over a sufficiently small time scale and regeneration effect prior to cavitation collapse are all important so laws of thermodynamics are not violated. Extra dimensions of gravity at short distances, if present, could also assist in the high energies required for the production of mini black holes in condensed matter. Such black holes lose mass very quickly through the emission of Hawking radiation observed in the form of explosion with mass-energy equivalence of  $E = mc^2$ . The possibility of tapping such explosive energy for peaceful purposes is discussed. As such it constitutes the third kind of atomic energy humans have entertained, the first two being fission and fusion.

## Introduction

Depletion of fossil fuels in the globe requires the development of alternate sources of green energy. The uranium resources used for the production of nuclear energy are also limited. Solar energy and windmills occupy ground space which is in scarcity. Although hydrogen provides a clean source of energy its production and storage are not easy. Controlled nuclear fusion still appears to be a far cry even after three decades of research due to difficulties in attaining and maintaining the required high temperatures and pressures. Fast-breeder nuclear reactors where nuclear fission occurs at

high neutron energies are meant to produce more fuel than they consume. Molten sodium metal is used as a coolant in fast reactors so as not to thermalize the neutrons poses a high risk of sodium-water explosive reaction in case of sodium leakage during reactor operation. Quite often, both during the cleaning of reactor components as well as during the decommissioning of sodium-cooled fast reactors, there exists a need to develop eco-friendly techniques to dissolve sodium peacefully. Liquid fire hydrants will also come in handy in case of major sodium fires. An attempt was by the author to develop a novel technique based on a 2M aqueous Epsom solution for dissolving peacefully sodium metal which ensures the peaceful (slow) release of



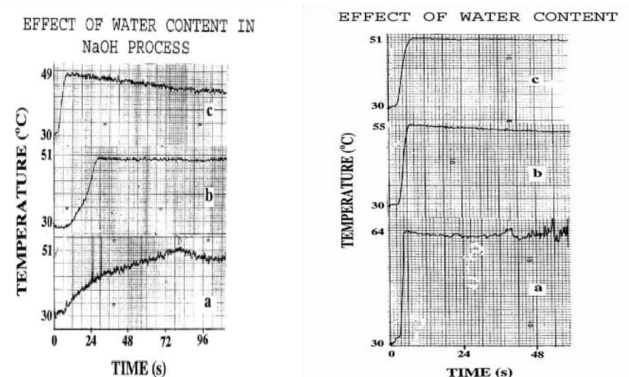
hydrogen generated during Na-H<sub>2</sub>O and Mg-H<sub>2</sub>O reactions into the atmosphere. In a dilute (0.85 M) Epsom solution, however, the hydrogen released exploded very violently causing the glass beaker containing the Epsom solution to evaporate completely indicating a very high temperature has been reached. The excess energy released was much more than the hydrogen combustion energy. The timing of the explosion has indicated the trapping of hydrogen in the solution and energy build-up prior to the explosion. Careful consideration of microscopic aspects of the nano crystalline structure formed by cavitation has indicated the possibility of high-energy collision of trapped protons due to oscillatory reactions involving steam cavitation (attractive) and electrostatic repulsion (repulsive). The energy released is consistent with the formation and instantaneous collapse of micro black holes which emit Hawking radiation. Such a process happening in condensed matter is observed for the first time in a laboratory and is shown to have certain advantages over that in the plasma state. Several obstacles are, however, to be overcome before such black hole production can be standardized as a commercial source of energy.

The novel method of nonviolent dissolution of sodium metal in a concentrated aqueous solution (2M) of Epsom salt (MgSO<sub>4</sub>·7H<sub>2</sub>O) was invented both for the removal of sodium from (i.e., cleaning) fast reactor components and petroleum industries as well as for sodium firefighting in case of an accident [1]. The cleaning process is an essential first step before the head operations of cooling and reprocessing of spent fuels in a fast nuclear reactor. The reaction of Na in 2M Epsom solution is mildly exothermic but the reaction products, namely Na<sub>2</sub>SO<sub>4</sub> and Mg(OH)<sub>2</sub> are eco-friendly (non-corrosive and non-hazardous), unlike the presently used caustic soda solution (10M of NaOH) method developed in France which releases caustic soda as a byproduct [2]. The sodium dissolution process in Epsom solution is peaceful as the reaction rate is slowed down by the presence of solute ions but the exact mechanism behind it is complex and will be described in detail. A bluish flame results on top of the glass beaker in which the dissolution was carried out due to the controlled release of hydrogen gas and its ignition with oxygen present in the air. This work has evoked considerable interest globally and has caught the attention of France who are the pioneers in fast nuclear reactors [2]. Realizing the importance of this work, the effect of mineral salts on the kinetics of the salt water reaction is currently being studied at the CEA Cadarache, France. The carbonation (water mist and carbon dioxide flow) process or a slow immersion of French SUPERPHENIX reactor fuel assemblies in pre-water used by them for cleaning sodium was not very effective. The presence of caustic soda associated with a high temperature may induce crack corrosion. The challenging performances of the new sodium fast reactor (SFR) fuel assembly cleaning require respecting several criteria: such as - a simplification of the process, - an improvement of safety, - a decrease in the duration of the operations, and - greater flexibility allowing for treatment of any type of fuel assembly. In this regard, an extensive citation of our innovative work [1] can be found in their paper [2]. The above novel study has evoked interest in South Korea as well due to their keen interest in the development of fast reactor technology [3,4]. Disposal of waste

sodium from sodium-wetted components used in fast reactors was successfully carried out at IGCAR, Kalpakkam by spraying concentrated (1.4 - 2M) Epsom solution [5]. Sodium draining from cold traps supplied by M/s Tamilnadu Petro Products Ltd was also successfully carried out by this process [6].

## Mechanism of peaceful dissolution of sodium metal in 2M Epsom solution

Textbook chemistry states that sodium metal in an aqueous solution will react with water and water only in any condition and that the salt added just reduces the thermodynamic activity of water. However, experiments carried out with different salt solutions under stirring conditions (without stirring sodium metal floats in aqueous solutions, and on exposure to air, sodium fire results which makes peaceful dissolution impossible) have shown that this theory holds only with NaOH solution in which the salt exists as ions at all concentrations even during sodium dissolution under stirring conditions. Detailed studies with Epsom (MgSO<sub>4</sub>·7H<sub>2</sub>O) solution, however, indicate a new dissolution mechanism based on cavitation induced *meta stable* crystals to be operative under stirring conditions. In this process, the sodium metal dissolves into the metastable cavitation-induced nano MgSO<sub>4</sub> crystals and expels Mg ions into water thereby forming Mg(OH)<sub>2</sub> and Na<sub>2</sub>SO<sub>4</sub>. Due to the following reasons, this mechanism was preferred over the conventional one in which sodium reacts with water to produce NaOH and hydrogen, and NaOH in reaction with MgSO<sub>4</sub> produces Mg(OH)<sub>2</sub> and Na<sub>2</sub>SO<sub>4</sub>. Two major differences observed between sodium metal dissolution in NaOH solution and in Epsom solution were: 1) In the NaOH solution, Na dissolution time *decreases* as water content *increases* since Na basically reacts with water (Figure 1, left). An enhanced NaOH concentration considerably slows down the dissolution process (sodium keeps on rotating for several minutes before complete dissolution) since with increasing salt concentration, water molecules are surrounded by more and more Na and OH ions from the NaOH salt added, thereby reducing the thermodynamic activity of sodium metal with water. However, no such Na dissolution time dependence was seen with the Epsom solution as shown in Figure 1 (right). The dissolution process with Epsom solution proceeds always fast. In fact, with increased Epsom concentration, the dissolution proceeds more rapidly



**Figure 1:** Variation of sodium metal dissolution time with water content. NaOH process (left), Epsom process (right). Water content increases from a to c in both figures. For more details refer to [1].



since basically, Na dissolves into Epsom crystal lattice formed by steam cavitation. 2) With NaOH solution, sodium solubility was found to *decrease* with *increasing* salt concentration since sodium reacts mainly with water. However, the amount of sodium that could be dissolved peacefully was found to *increase* with Epsom salt content since sodium basically dissolves into  $\text{MgSO}_4$ . These results clearly showed the inapplicability of conventional chemistry rules under cavitation conditions.

### Hydrodynamic cavitation and Steam bubble cavitation

Hydrodynamic cavitation in fluid mechanics and engineering normally refers to the phenomenon in which the static pressure of a liquid reduces to below the liquid's vapour pressure, which occurs in places where the liquid has been accelerated to high velocities, as in the operation of centrifugal pumps, water turbines, and marine propellers leading to the formation of small vapour-filled cavities in the liquid. When subjected to higher pressure, these cavities, called "bubbles" or "voids", collapse (implode) and can generate shock waves that may damage machinery. These shock waves are strong when they are very close to the imploded bubble, but rapidly weaken as they propagate away from the implosion. Hence cavitation is in general considered as a destructive phenomenon that can have a profound impact on the integrity of materials, particularly in steam systems. This article, in contrast, highlights the beneficiary aspects of cavitation namely the promotion of crystal growth. A strong correlation exists between crystal nucleation and cavitation [7].

Figure 2 shows pictures of Epsom solution in a glass beaker used for sodium dissolution. On vigorous stirring (1000 rpm), the solution turns turbid (milky) because of entrained air bubbles. The cavitation bubbles are, however, invisible in turbid conditions. There is no air but vapor of the liquid is present in the cavitation bubbles. As the water heats up, the solubility of the gas decreases, causing the air bubbles to rise to the surface. Bubbles that form when water boils are made up of water vapour. The application of Bernoulli's equation in this experiment, however, showed that reduced pressure during stirring is insufficient to cause *hydrodynamic cavitation* even under high-speed stirring as the blade size of the stirrer used is small (~cm) unlike those of ship propellers (~m). However, the high exothermicity arising during sodium metal dissolution and the distribution of these local boiling spots during stirring can cause *steam cavitation* [8]. After all, a fluid can vaporize

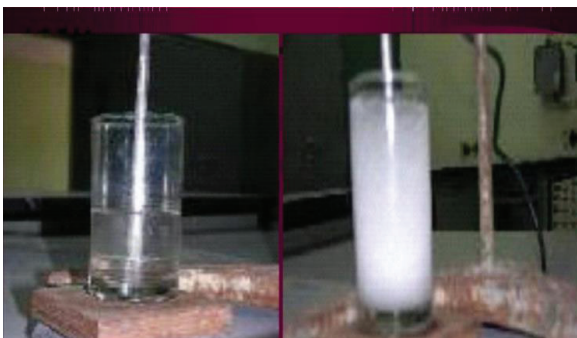
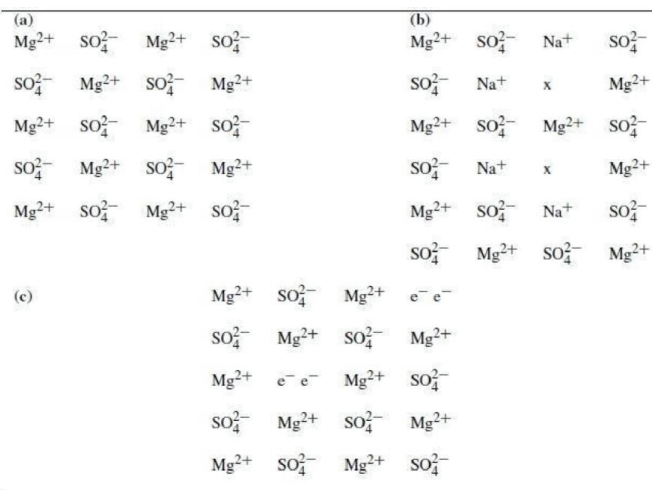


Figure 2: Epsom solution (2 M) in a glass beaker under stationary (left) and under external stirring (right, ~1000 rpm) conditions.

when the temperature is too high or ambient pressure becomes too low. The process of generation, subsequent growth, and collapse of the cavitation bubbles results in high energy densities, resulting in high temperatures and pressures at the surface of the bubbles but only for a very short time. The crystal formation due to cavitation has been attributed to a number of factors such as i) local temperature increase, ii) pressure changes leading to rapid cooling rates ( $10^7$ - $10^{10}$   $\text{k.s}^{-1}$ ), iii) concomitant shock waves, which will overcome energy barriers to nucleation and promote crystal nucleation and its growth even at a modest supersaturation level [9]. We extend this fact for the first time to explain the formation of meta stable crystalloids due to exothermicity arising during sodium metal dissolution in concentrated but under-saturated aqueous Epsom salt solutions used for peaceful sodium metal dissolution.

### Peaceful dissolution process of sodium in concentrated (2M) Epsom solution

A recent study shows that if a steam bubble is introduced in a cold water pool, due to a fast condensation of the steam from the surface of this bubble, a phenomenon similar to the cavitation collapse of the bubble occurs. In fact, the steam bubble cavitation process was found to be more efficient compared to conventional cavitation processes because of the elimination of the intermediate energy-interchange process. The word "meta" is used here because the crystals once formed by cavitation in under-saturated solutions get dissolved subsequently during rotation of the solution and again get reformed due to cavitation and so on. Unlike the case with super-saturated solutions, the tiny (nano) crystals formed in under-saturated solutions cannot be recovered but their repeated formation and collapse due to the exothermic reactions during sodium dissolution can drastically change the rates of chemical reactions involved. Specifically, as a result of these metastable crystal formations, while the  $\text{Na-H}_2\text{O}$  reaction is suppressed, Na incorporation into Epsom crystals is promoted by Na-Mg exchange reaction as indicated in Figure 2b. Only a portion of Mg released during this reaction reacts with water immediately while the rest gets into the metastable crystal lattice formed during exothermic reactions again and again before completely reacting with water thereby forming  $\text{Mg(OH)}_2$  and  $\text{Na}_2\text{SO}_4$  as reaction products. This is how the sodium dissolution rate is controlled. The results are, however, consistent with the interpretation that sodium is dissolved primarily in the cavitation-produced nanocrystals of  $\text{MgSO}_4$  rather than reacting with water whereas in NaOH solution, sodium primarily reacts with water similar to the case with pure water except for the fact that the reaction rate in the NaOH solution is reduced due to the presence of solute molecules and therefore no explosion results during sodium dissolution in concentrated (10M) NaOH solution. However, a high molarity of NaOH is required for this action. As seen in Figure 1, Na addition in the salt solution increases the solution temperature in both cases since both  $\text{Na-H}_2\text{O}$  and  $\text{Mg-H}_2\text{O}$  reactions are exothermic. We can visualize sodium incorporated  $\text{MgSO}_4$  crystal in a 2M solution as a bimetallic crystal. As shown in Figure 3, its simplified structure before Na



**Figure 3:** A simplified picture of the Epsom salt solution before (10MgSO<sub>4</sub> - (a)) and after sodium dissolution (8MgSO<sub>4</sub>.2Na<sub>2</sub>SO<sub>4</sub> - (b)) in the 2 M standard solution under vigorous rotation recommended for sodium disposal. Exchange of Na with Mg takes place by 2Na–Mg exchange reaction in cavitation-induced nanocrystals resulting in anion vacancies (marked as x). Dissolution of Mg in MgSO<sub>4</sub>(c).

dissolution can be depicted as 10MgSO<sub>4</sub> and after Na dissolution as 8MgSO<sub>4</sub>.2Na<sub>2</sub>SO<sub>4</sub>. These are metastable crystals that are difficult to photograph, especially under turbid conditions. The subsequent dissolution process of Mg atoms in metastable MgSO<sub>4</sub> involves the incorporation of Mg into the lattice in the form of Mg<sup>2+</sup> ions and electrons trapped at anion vacancies (Figure 3c) and its release into solution repeatedly results in the formation of Mg(OH)<sub>2</sub>. Such an additive reaction slows down the sodium dissolution process and that is how Na dissolution proceeds peacefully in 2M Epsom solution. But for such an exchange followed by additive reactions, the sodium dissolution process would have been violent similar to the sodium–water reaction in pure H<sub>2</sub>O [10] or in some other salt solution such as NaCl. In NaCl solution, the sodium exchange or additive reaction discussed above does not take place efficiently and the reaction turns violent accompanied by Na fire.

### Explosive dissolution of sodium metal in a dilute (0.85M) aqueous Epsom solution

When a reactor scientist suggested cutting down the salt concentration to save cost, a 0.85 M Epsom solution was chosen to dissolve Na which unexpectedly resulted in an intense explosion accompanied by a shock wave and vaporization of the Borosil glass beaker containing the salt solution on the completion of sodium dissolution, i.e., nearly 30 s after Na addition. Presence of Mg(OH)<sub>2</sub> – a white milky solution – though in dilute concentration when compared to the more viscous solution observed in concentrated Epsom solution in this time scale was seen through the glass beaker prior (this is usually taken as an indication of completion of sodium dissolution in the solution) to the explosion. After the intense explosion, there was Na aerosol in the air with a pungent smell everywhere in the room despite the functioning of an industrial exhaust fan which indicated the vaporization of the solution containing sodium. No trace of the glass beaker was seen although the stainless steel stirrer continued to rotate indicating that the explosive energy was released outwardly.

The sound and shock waves were deafening which attracted the attention of occupants of the huge two-storied building where this experiment was carried out. Since the dissolution in this case was carried out outside the fume hood no material damage occurred. Minute molten glass needles, however, flew from the glass beaker all around causing invisible holes in plastic water bottles making them inoperative and a minor injury (glass needle scratch) in the hand of the author who was standing 4 meters away indicating that Pyrex glass has melted and the temperature rise in the system has crossed 1000°C. At high Epsom concentration (>1.6 M), 2Na–Mg cation exchange reaction leads to the peaceful dissolution of sodium in Epsom solution in about 30s. At low Epsom concentration (<0.7 M), a Na–H<sub>2</sub>O reaction occurs leading to an explosion within about 5s. In pure water, sodium addition was found to cause instant explosion (μs) under stirring conditions. At 0.85 M Epsom concentration both the reactions mentioned above are shown to take place with equal probability [11,12]. The delayed explosion has clearly indicated that *primaface the hydrogen gas released during sodium dissolution has somehow got trapped in the salt solution before explosion* – the mechanism of which is the subject matter of the present paper. So it was an accidental discovery (serendipity)! This means that instead of a 2Na–Mg cation exchange, a 2H–Mg exchange seems to have taken place in the 0.85 molar Epsom solution during sodium dissolution. A cage meant to trap a massive elephant (Na<sup>+</sup> ions) instead trapped the elusive panthers (H<sup>+</sup> ions). Towards the end of sodium dissolution in the 0.85 M Epsom solution, i.e. ~30s from the start, the original aim of dissolving sodium in a dilute aqueous solution of Epsom salt at room temperature appears to have been almost achieved at less cost than in the standard solution but the dissolution was not peaceful. *Preliminary calculations have shown that the energy released from the explosive solution is far excess of hydrogen combustion energy in the air.* This experiment was successfully repeated a few times though the energy release in each experiment was not quantitatively estimated.

### Hydronium ion (H<sub>3</sub>O<sup>+</sup>) and Aquodium (H<sub>4</sub>O)<sup>2+</sup> ion (Figures 4,5)

Figure 5 In a dilute Epsom solution, MgSO<sub>4</sub> crystal on trapping Na<sup>+</sup> ions gets converted to Na<sub>2</sub>SO<sub>4</sub> crystal through Mg<sup>2+</sup> → 2Na<sup>+</sup> exchange reaction. Metastable H<sub>2</sub><sup>2+</sup> species get trapped at a cation site in Na<sub>2</sub>SO<sub>4</sub> crystal through 2H–Mg exchange. Water (H<sub>2</sub>O) molecules occupy the adjacent anion site to donate two lone pair electrons to the two hydrogen ions in the H<sub>2</sub><sup>2+</sup> species so as to stabilize the H<sub>2</sub><sup>2+</sup> ion at the cation site beside a water molecule trapped at a neighboring anion site as shown.

Hydronium ion (H<sub>3</sub>O)<sup>+</sup> shown in Figure 4 is formed in liquid state as a result of a coordinate covalent bonding (also known as dative bonding) of a hydrogen ion with one of the lone pairs of electrons of oxygen in water situated in the neighboring lattice site. The above reaction takes place with the hydrogen ions released during the mixing of a strong acid with water, is strongly exothermic, and has an important bearing in the proposed energy release process. Usually, in solution state, the second lone pair of electrons of oxygen in the H<sub>2</sub>O molecule

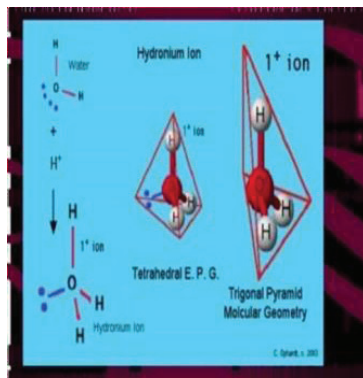


Figure 4: Hydronium ion ( $\text{H}_3\text{O}^+$ ).

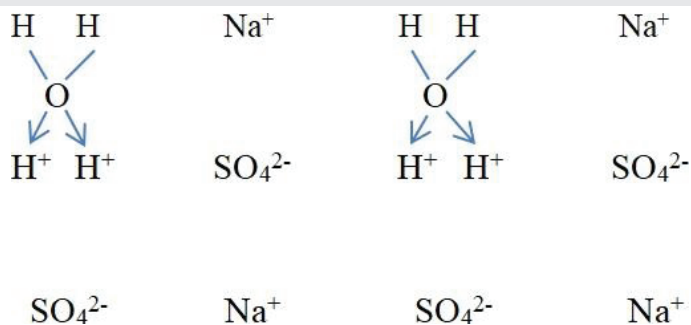


Figure 5: In a dilute Epsom solution,  $\text{MgSO}_4$  crystal on trapping  $\text{Na}^+$  ions gets converted to  $\text{Na}_2\text{SO}_4$  crystal through  $\text{Mg}^{2+} \rightleftharpoons 2\text{Na}^+$  exchange reaction. Metastable  $\text{H}_2^{2+}$  species get trapped at a cation site in  $\text{Na}_2\text{SO}_4$  crystal through  $2\text{H}-\text{Mg}$  exchange. Water ( $\text{H}_2\text{O}$ ) molecules occupy the adjacent anion site to donate two lone pair electrons to the two hydrogen ions in the  $\text{H}_2^{2+}$  species so as to stabilize the  $\text{H}_2^{2+}$  ion at the cation site beside a water molecule trapped at a neighboring anion site as shown.

does not form a dative bond with another hydrogen  $^2$  ion possibly due to the electrostatic repulsion between the two hydrogen ions. However, in other systems, such a bonding which results in the species  $(\text{H}_4\text{O})^{2+}$  – diprotonated hydronium, ion is known to exist [13,14]. In the explosive solution, for charge balance, the second lone pair electron in the  $\text{H}_2\text{O}$  molecule is also anticipated to form a second dative bond with another hydrogen ion to form a  $\text{H}_2^{2+}$ , a diprotonated hydronium ion also known as Aquodium ( $\text{H}_4\text{O})^{2+}$  ion with the assistance of lattice energy. It is anticipated that  $\text{H}_2^{2+}$  enters along with the  $\text{H}_2\text{O}$  molecule and exits the crystal as one species since  $\text{H}_2^{2+}$  is highly unstable. The species,  $\text{H}_4\text{O}^{2+}$  (Aquodium ( $\text{H}_4\text{O})^{2+}$  ion) is capable of stabilizing two hydrogen ions at a  $\text{Mg}^{2+}$  cation site in a  $\text{MgSO}_4$  crystal lattice site momentarily and is proposed as the precursor in the energy production process. The crystal lattice energy should overcome the repulsive force between the two hydrogen ions attached to the  $\text{H}_2^{2+}$  species at least momentarily. This bonding plays a pivotal role in the spatial confinement of the hydrogen ions surrounded by electrons in cavities (point defects formed by ion exchange or displacement reactions) in the proposed metastable nanocrystals. Three factors favor the metastable crystal structure proposed in Figure 5. One is the charge compensation. i.e., the incorporation of  $\text{H}^+\text{H}^+$  ion species stabilized with two electrons from an  $\text{H}_2\text{O}$  molecule forming two dative bonds which bring the two hydrogen ions together at one  $\text{Mg}^{2+}$  ion site momentarily as shown in Figure 5. Thus the cation charge balance i.e.  $\text{H}_2^{2+}$  in lieu of  $\text{Mg}^{2+}$  is achieved in the

ionic crystal. Second is their size. The  $\text{H}_2^{2+}$  species with the two electrons resembles an  $\text{H}_2$  molecule whose interionic distance of 74 pm matches very well with the ionic radius of the  $\text{Mg}^{2+}$  ion which is = 72 pm. The third factor is the ionic size of  $\text{SO}_4^{2-}$  anion (230 pm) and  $\text{H}_2\text{O}$  molecule (275 pm) are nearly the same which makes their replacement idealistic, i.e., a water molecule can occupy a neighboring sulphate ion vacancy formed during crystal formation and participate in dative bonding as shown in Figure 5.

However, the instability of the crystalline structure due to increased electrostatic repulsion when the two protons are brought together by cavitation force will prevent the formation of big stable crystals. The electrostatic repulsive force between the two protons at closer distances should break the crystal the moment it is formed leading to the release of hydration energy in the salt solution.

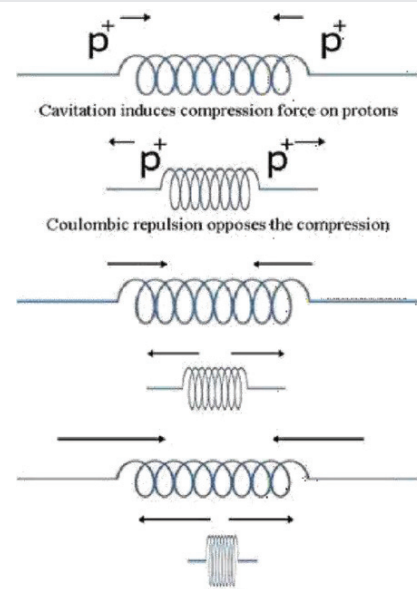
### Plausible mechanisms for explosion

The earlier claim made by Taleyarkhan, et al. [15] that the pressures and temperatures inside the collapsing cavitation bubbles could be high enough to initiate nuclear reactions has, however, been questioned since the bubble collapse is strongly cushioned and energy being dissipated by several factors may prevent the temperature from approaching levels required for detectable nuclear fusion [16]. *Per se*, it should be emphasized that the principle of cavitation is used by us in this study primarily only in the formation of metastable crystals and not in the production of high temperatures or pressures directly to produce energies required for any nuclear reactions. It will be shown that the latter process occurs through a CCRO reaction.

**Hydrogen trapping in metastable crystals:** The  $p-p$  nuclear fusion proposed earlier [17] also faced a conceptual hurdle. In the sun and stars nuclear fusion due to hydrogen, collision is a slow process arising from the fact that  $1p^1 + 1p^1 \rightarrow \text{D} + e^+ + \nu^- + 0.42 \text{ MeV}$  is a weak nuclear reaction involving the conversion of a proton to a neutron and hence is slow (sun burns for several years) while the observed explosion in Epsom solution is very fast. Instead other viable processes especially the evaporation of miniaturized black holes formed during high energy proton collisions in condensed matter in a confined space mimicking nuclear potential leading to nuclear-like explosions arising out of relativistic proton collisions/compression in condensed matter will be highlighted. Other theories proposed in this regard which include the volume Casimir effect and phonon amplification appear to violate the first law of thermodynamics i.e. the source of input energy is not properly defined [12].

**Cavitation and electrostatic repulsion:** It is seen that steam cavitation and proton repulsion oppose each other. The former force is attractive which brings the  $\text{Na}^{2+}$  and  $\text{SO}_4^{2-}$  ions along with the  $\text{H}_2^{2+}$  species (or more precisely  $\text{H}_4\text{O}^{2+}$  since the water molecule always accompanies the  $\text{H}_2^{2+}$  species) together to form tiny ionic crystals of  $\text{Na}_2\text{SO}_4$  (all  $\text{Mg}^{2+}$  ions are expelled from the crystal lattice due to  $2\text{H}-\text{Mg}$  exchange reaction at 0.85 M concentration!). The latter force involving  $\text{H}_2^{2+}$  species due to like charges is repulsive leading to the collapse of the crystal the moment it is formed. The divalent crystal lattice

energy demands the positioning of two protons at a single  $Mg^{2+}$  ion lattice site and hence the energy needed to overcome the electrostatic repulsion of the two protons bonded to the two lone pair electrons in the oxygen atom in a water molecule is provided by the ionic crystal lattice energy. The crystal structure shown in Figure 5 which utilizes several fascinating properties of water represents the precursor state of the explosive solution after Na dissolution. The oscillatory chemical reactions, however, repeatedly release hydrogen ions into the aqueous solution. Because the hydrogen ion is so tiny, a large amount of charge is concentrated in a very small area and the polar water molecules are strongly attracted to it thereby forming the  $H_3O^+$  ions. This “hydration” of the hydrogen ion involves not only the formation of a coordinate covalent bond to one of the water molecules but a large number of strong hydrogen bonds, so it is a strongly exothermic process ( $H_3O^+$  hydration energy = 461.1 kJ/mol). The formation of  $H_4O^{2+}$  would release much more energy. *Such local hot spots promote crystallization.* However, stirring distributes uniformly the heat energy released from local hot spots. Therefore, the rise in bulk solution temperature in the beaker is visualized to be not more than 60–70°C until the nuclear-related energy production reaction commences. Dean Sinclair who is the proponent of oscillator/substance theory [18], is of the opinion that the formation of a symmetric para-hydrogen (nuclei with opposing spin) is a necessary pre-requisite for the two hydrogen ions to come closer. This has been presumed to occur when the rotations of both protons are aligned on the same vector, in exactly opposite senses such that the two rotations will cancel before collision occurs. In this species, the electron density states get automatically squeezed as they lie in between the protons so the CCRO process can squeeze the two protons together. This approach brings in the importance of nuclear spin in micro black hole formation during p-p collision, a feature more likely to happen in the solid state rather than in the plasma state. The time delay between sodium dissolution (10 to 15 s after the sodium addition) and the actual explosion witnessed 30s after the sodium addition in 0.85 M Epsom solution might be due to the time taken for the generation of the precursor to initiate the chain reaction which appears to be the very slowest, least probable, step of the whole process. Once the precursor state is set, the energy production reaction subsequent to high energy collision of protons in the hydrogen molecular ion species due to the CCRO process depicted in Figure 6 is anticipated to be very swift, almost instantaneous. Before the hydration energy released is dissipated into the bulk of the solution, further build-up of hydration energy should take place or else there is a chance of energy dissipation from the nanocrystal to the bulk of the solution which would slow down the reformation of the crystal and the conditions required for high energy proton collisions would not have been achieved. The extent of the regeneration suggested is important in this context. Regeneration of a small crystal over a sufficiently small time scale could, in principle, occur. The opposite limit of a large crystal being destroyed and it being regenerated, after a long period of time, involves a significant reduction in entropy in a situation associated with an explosion that simply cannot occur. Therefore, details, involving the crystal size, time scale, and the mechanism for



**Figure 6:** Spring-like action between two protons trapped in cavities due to oscillations caused by two opposing forces – one by cavitation and the other by coulombic repulsion. Note with an increasing number of oscillations the distance between the protons decreases. Eventually, they would collide at relativistic speeds. This is a simplified 2-dimensional picture. In a crystal, pressure is applied 3-dimensionally which squeezes the  $H_2^{2+}$  species three-dimensionally.

creating the nanocrystal regeneration effect prior to cavitation collapse are all important so laws of thermodynamics are not violated.

$H^+$  ions are the smallest ions known and are more difficult to trap in a crystal lattice but two such ions brought together in a divalent metal lattice make their trapping at least momentarily a reality. The electrostatic force of repulsion between the two hydrogen ions would, however, prevent them from coming together thereby making it a highly unstable structure. It would hence tend to break the moment it is formed, at a much faster rate than the normal dissolution process of cavitation-induced crystals formed during Na dissolution in a 2M solution. However, cavitation induced by the repeated release of hydration energy will reform these crystals quickly with more vigor, and thus an oscillatory reaction sets in. The situation is akin to a mechanical spring being pushed inward by an external force. The more force that is applied (due to cavitation) the closer the protons approach and the more its recoil force (due to electrostatic repulsion) as depicted in Figure 6. *In other words, the electrostatic repulsive force between protons which has been a stumbling block in bringing the two protons close together is now shown to help the two protons to come together with the assistance of cavitation force with increasing speed as the number of oscillations progresses.* Basically, the above repulsive force helps in the regeneration of nanocrystals rapidly. The two strong opposing forces mentioned above would never allow the crystals to grow to larger dimensions. This is the premise for assuming the proposed proton-proton collisions in metastable nanocrystals resulting in Cavitation- Columbian Repulsion oscillation (CCRO).

**High energy p-p collisions and micro black holes:** According to the general theory of relativity, a black hole should form whenever some mass is squeezed into a very small

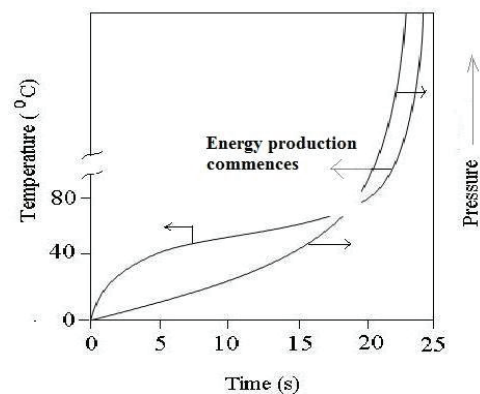


region of space. Two protons moving in opposite directions do have an invariant kinetic energy associated with the whole system. Therefore, during a head-on collision of protons, a high concentration of mass would result, and a mini black hole could form through quark interaction. But the detector at large hadron collider (LHC) at CERN has searched for such events without success amongst all the proton-proton collisions recorded during the 2010 LHC running at 7 TeV centre-of-mass energy (3.5 TeV per proton beam). A computer model showed conclusively for the first time that a particle collision really can make a black hole if it collides with an energy above one-third of the fundamental limit called the Plank energy ( $1.22 \times 10^{28}$  eV =  $1.22 \times 10^{16}$  TeV) [19,20]. Thus creating a microscopic black hole would require billions of times more energy than the LHC can produce. Furthermore, the  $p$ - $p$  collision cross section is very small just 0.07 barns at the LHC energies, so most of the protons do not hit their partners but continue to move in their original direction. Black holes with a mass that is extremely small are extremely hot and unstable. Stephen Hawking [21] developed a theoretical argument for its existence in 1974. The radiation temperature, called *Hawking temperature*, is inversely proportional to the black hole's mass, so micro black holes are predicted to be larger emitters of radiation than larger black holes and should dissipate faster their mass. When formed, micro black holes would evaporate nearly as soon as they are created ( $< 10^{-25}$  s) with the emission of Hawking radiation which consists of a sudden blast of a few energetic particles and heat [22]. The uncertainty principle between momentum and position ( $\Delta p \cdot \Delta x \sim h/2\pi$ ) requires that to pinpoint the position of protons so as to increase their collision probability, the uncertainty in momentum of the protons becomes infinite. Increasing the density of the proton beams will result in their coulombic repulsion thereby the high-energy protons cannot be focused for collision.

Therefore the argument arises that if the protons are confined in space (i.e., minimized  $\Delta x$ ), their uncertainty in momentum ( $\Delta p$ ) becomes exceedingly large—so large that no crystal lattice (metallic or ionic!) can hold and hence high energy proton collisions leading to nuclear reactions can take place only in plasma state. It will, however, be seen that it is possible to achieve high energy proton collisions through simpler routes momentarily in the condensed matter without violating the principles of quantum mechanics or uncertainty principle.

While proton collision events with high energy are extremely rare due to energy constraints in particle accelerators, this type of collision has been going on for literally billions of years in the earth's atmosphere by the collision of ultrahigh-energy cosmic rays with nuclei of oxygen, carbon, nitrogen and other elements present in the atmosphere. Since the earth has not (yet) disappeared into one of these black holes, the much less massive manmade mini black holes should be quite safe. As stated above, it is unique that there is no upper limit to the number of proton oscillations and the energy build-up in the proposed CCRO process. This means the oscillation speed of protons and hence their kinetic energies can increase exponentially or even instantly with the number

of oscillations due to phonon amplification to any desired extent at ultrahigh speed until the endpoint (micro black hole formation) is reached with minimum energy input. Thus the achievement of proton energies of hundreds of TeV required for the production of mini black holes is very well possible in condensed matter involving plasma state within a very short interval of time. The two protons in the  $H_2^{2+}$  species are confined in a small space at a single crystal cation lattice site in the  $Na_2SO_4$  nanocrystal which increases the head-on collision probability at high energies, unlike the collision probability at particle accelerators which involve random collisions. Since CCRO oscillations of protons depicted in Figure 5 around the divalent cation site precede micro black hole formation, with the speed of oscillations increasing exponentially first and in the final stages (preceding fusion) almost instantaneously shooting up as shown in Figure 7, the protons undergo a very large momentum ( $p$ ) associated with a large  $\Delta p$  leading to a low value of uncertainty in its position, i.e.  $\Delta x$  at an incredibly short internal time, thereby satisfying the relation  $\Delta p \cdot \Delta x \sim h/2\pi$ . Heat energy transfer from lower temperature to higher temperature envisaged in this work would invoke a violation of the second law of thermodynamics though in principle in small systems in short intervals of time, such violations have been experimentally shown to occur. Wang and his colleagues dragged a micron-sized bead through a container of water using optical tweezers and found that, on occasion, the water molecules interacted with the bead in such a way that energy was transferred from the liquid to the bead [23]. These additional kicks used the random thermal motion of the water to do the work of moving the bead, in effect yielding something for nothing. For periods of movement lasting less than two seconds, the bead was almost as likely to gain energy from the water as it was to add energy to the reservoir. No useful amounts of energy could, however, be extracted from the set-up because the effect disappeared if the bead was moved for time intervals greater than two seconds. An international team of researchers from Barcelona, Zurich, and Vienna found that a nanoparticle trapped with laser light temporarily violates the famous second law of thermodynamics, something that is impossible on human time and length scale [24]. Their results



**Figure 7:** The conceptual temperature and pressure envisioned on the hydrogen ion species ( $H_2^{2+}$ ) trapped in the metastable nanocrystal during sodium metal dissolution in a dilute aqueous Epsom solution (0.85 M). These are conceptual and not measured values since the solution actually explodes at the end of the sodium dissolution process.



are also in good agreement with predictions of the “fluctuation theorem”, a theory developed at ANU 10 years ago to reconcile the second law with the behavior of particles at microscopic scales. “The results imply that the fluctuation theorem has important ramifications for nanotechnology and indeed for how life itself functions”, claim the researchers [25]. At atomic scales over very short periods of time, statistical mechanics is pushed beyond its limit, and the second law does not apply. Violation of the second law of thermodynamics by small ensembles of particles within larger systems has been predicted a decade ago [26]. But what Wang, et al. [23] report is that even larger systems of thousands of molecules can also undergo fleeting energy increases that seem to violate the venerable law. In 1878, the physicist James Clerk Maxwell wrote in a book review for Nature: The truth of the second law is ... a statistical, not a mathematical, truth, for it depends on the fact that the bodies we deal with consist of millions of molecules. Hence the second law of thermodynamics is continually being violated, and that to a considerable extent, in any sufficiently small group of molecules belonging to a real body. For larger systems over normal periods of time, however, the second law of thermodynamics is absolutely valid.

A parallel to the proposed ultrafast energy build-up in the CCRO process leading to the formation and subsequent evaporation of micro black holes can be found in Yukawa's proposal for strong nuclear force. Yukawa proposed that a *pion* is exchanged between nucleons, transmitting the force between them [27]. The pion has mass and can only be created by violating the conservation of mass-energy. This is allowed by the Heisenberg uncertainty principle (HUP) if it occurs for a sufficiently short period of time. The HUP relates the uncertainties  $\Delta E$  in energy and  $\Delta t$  in time by  $\Delta E \Delta t \geq h/4\pi$ , where  $h$  is Planck's constant. Therefore, the conservation of mass-energy can be violated by an amount  $\Delta E$  for a time  $\Delta t \approx h/4\pi \Delta E$  in which time no process can detect the violation. This allows the temporary creation of a particle of mass  $m$ , where  $\Delta E = mc^2$ . The larger the mass and the greater the  $\Delta E$ , the shorter the time it can exist. This means the range of the force is limited because the particle can only travel a limited distance in a finite amount of time. In fact, the maximum distance is  $d \approx c \Delta t$ , where  $c$  is the speed of light. The pion must then be captured and, thus, cannot be directly observed because that would amount to a permanent violation of mass-energy conservation. Such particles (like the pion above) are called *virtual particles* because they cannot be directly observed but their *effects* can be directly observed. Realizing all this, Yukawa used the information on the range of the strong nuclear force to estimate the mass of the pion, the particle that carries it. Making parallel, if the energy build-up during the CCRO process is sufficiently fast, the achievement of proton energies required for microblackhole formation in condensed matter is very well possible before cavitation collapse.

Furthermore, at shorter distances, the quantum theory of gravity predicts that gravity could have higher dimensions and the gravitation force of attraction would increase much faster than the inverse square law which would assist in enhancing collision energy.

It is stated that if micro black holes do appear in the collisions created by the large hadron collider (LHC), they would disintegrate rapidly, in around  $10^{-27}$  seconds. They lose mass through the emission of Hawking radiation with mass energy equivalence of  $E = mc^2$  and thus the first law of thermodynamics holds. This is the premise for the energy release mechanism through the proposed proton-proton collisions in metastable *nanocrystals*. Thus the solution to the present energy crisis lies in nanotechnology. The nanocrystals are metastable because we are dealing with an under-saturated solution. Stable and larger-sized crystals can be formed only in an over-saturated solution. After the cavitation collapse, an initial burst of energy would be released from the matter as it reaches some type of thermodynamic and spatial equilibrium. Entropy, temporarily held within the cavity is also released back to its surroundings so the second law of thermodynamics also holds. In this case, we have a “hot condensate” due to the extreme pressures and densities created within the void collapse. When that external pressure is relieved, the condensate may “flash”, releasing Hawking Radiation as it seeks a new thermodynamic and spatial equilibrium. If it remains trapped within the compressed void it may just slowly emit lower levels of hawking radiation as it slowly devours any nearby matter. Perhaps the CCRO mechanism is the only process that could lead to a sudden burst of energy.

## Summary

For the first time, a viable process capable of the production of miniaturized black holes in mother Earth (with ordinary water as the fuel and sodium metal as a catalyst) releasing excess energy on their evaporation has been visualized. The electrostatic repulsive force between protons which has been a stumbling block so far in achieving their collision is now turned in its favor by exploiting the fascinating properties of water. This proposal should be followed up by detailed experiments to tap this green energy for useful purposes. The total binding energy released in the fission of a  $^{235}\text{U}$  nucleus averages about 200 MeV or  $3.2 \times 10^{-11}$  J. The energy released by the nuclear fusion of two hydrogen atoms is about  $7.24 \times 10^{-19}$  J. The energy released during the evaporation of a black hole containing two protons would be about  $9.22 \times 10^{-14}$  J. Thus the mass energy conversion is most efficient in the case of micro black holes. Therefore, a black hole power plant (BHPP) is envisioned as a machine that uses a series of black holes to convert mass into energy for useful work. As such it constitutes the third kind of atomic energy humans have entertained, the first two being fission and fusion. Unlike the other two, the BHPP is a green form of energy as no radioactive by-products are envisaged since high-energy proton collisions emit only Hawking radiation. Their decay would result in a sudden blast of a few energetic particles and heat. Quantitative measurement of the type of radiation, their spectra, and energy released on micro black hole evaporation should be measured. Tapping such explosive energy for peaceful purposes would, however, require a lot of caution. Several obstacles are to be overcome before such black hole production can be standardized as a commercial energy source. The first is to eliminate the interference of the Na-H<sub>2</sub>O reaction which is basically chemical in nature or





else a premature explosion with a weaker intensity will occur. As the container holding the Epsom solution also evaporates during the evaporation of micro black holes, its effect should be studied in detail. A sample size of each BHPP plant also requires to be standardized. Too big a sample size would release an uncontrollable amount of energy while too small a sample size would require frequent recharging. Taking an optimistic view, it looks like the time to move from computer modeling to experimental design of atomic reactors based on black holes is not far away.

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